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最佳奢侈稅政策與其退出機制

The Optimal Luxury Tax Policy and Its Exit Mechanism

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摘　要

在歷經全球金融風暴及歐債危機後，多數政府採取了撙節預算的方針來因應。儘管課徵奢侈稅是增加政府稅收的另一好方案，但該政策卻也可能衍生負面的消費外部性及扭曲的經濟行為。除對奢侈稅負向的影響經濟成長及資本存量之理論推論外，本研究亦推導出奢侈稅之退場機制的最佳啟動時間點。另外，本研究亦提供台灣之理論奢侈稅率之模擬。相信本文研究能提供政府奢侈稅方面的政策參考。

關鍵詞：奢侈品、最佳稅率政策、奢侈稅、退場機制、經濟成長

ABSTRACT

After experiencing the global financial crisis and the European debt crisis, most governments have tended to cut their budgets. Although imposing a tax on luxury goods is a good way to increase government revenue, it also induces negative consumption externalities and thereby leads to a possible distortion in economic behavior. In addition to a theoretical demonstration of the negative influence of a luxury tax on economic growth and the capital stock, this paper, more importantly, theoretically deduces the timing for starting the exit mechanism of the luxury tax, which is related to several economic conditions. In addition, we simulate the optimal luxury tax for Taiwan. Our results should serve as valuable reference to policy-makers.

Key words: luxury goods, optimal tax policy, luxury tax, exit mechanism, economic growth

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1. Introduction

After experiencing the global financial crisis and the European debt crisis, most governments tend to cut their budgets so as to mitigate the harm of the global disaster. Despite the way of charging luxury goods can raise a government's revenue, it can also accompany with a negative influence on economic growth. There is a well-known case in the United States. The U.S. government enacted a luxury tax in November 1990. Meanwhile, consumers had to pay an additional tax for luxury goods-such as private yachts, planes, furs, jewelry, and luxury cars - when prices of luxury goods were over the regulated level. Yet, in August 1993, the U.S. Congress decided to end the imposed luxury tax because the received tax revenue did not meet the original expectation. More importantly, the livelihoods of the suppliers and workers of producing those luxury products all suffered severely, since the sales of related luxury goods were decreasing due to the flexible substitution between those luxury goods for the rich (Browning & Crossley, 2000). As a result, the luxury tax was annulled and replaced by the 1993 Revenue Reconciliation Act.

Taxation theory suggests that necessity goods (i.e., the price elasticities of demand are low) should be taxed with a higher level, whereas luxury goods with higher price elasticities should be taxed lower.[1] However, this is not so in practice. Most countries impose higher tax on luxuries but lower tax on necessities so that the poor can afford necessity goods without much additional tax payment (e.g., Deng & Ng, 2004). As a whole, searching for the optimal tax rate becomes one of the primary lessons for policymakers.

As argued in Veblen (1922), a commodity price is determined not only by its own character, but also by its package and environmental situation. Any goods of satisfying such a characteristic are called Veblen goods, and the effect is called the Veblen effect. By definition, when the Veblen effect occurs, if the willingness of paying for the goods is increasing with its price (i.e., the known phenomenon "conspicuous consumption"), commodity sales will be better with increasing commodity price (e.g., real estate issue, see, for example, Velma & Swarn, 2011; Turnbull et al., 2006). Conspicuous consumption has also been associated with the existent Veblen effects. Note that one type of luxury goods is regarded as Veblen goods. In addition, luxury goods possess high income elasticity, but the supply of luxuries is quite inelastic so that levying a luxury tax may hurt suppliers or their employees, rather than the rich as the case in the U.S. we have discussed above.

Generally speaking, a tax reform or executing a luxury tax can raise government revenue, thereby suppressing over-priced assets, moderating inequality (e.g., the inequality in terms of competitiveness among sports teams) as mentioned by Dietl et al. (2010), and narrowing the income gap between the rich and the poor; however, it can also lead to more deadweight loss on an economy's social welfare. Therefore, policymakers need to consider an appropriate rate of luxury tax to balance its triggered advantages and disadvantages.[2] Put differently, the government is better off to set an exit mechanism for luxury tax so that it can improve or stabilize economic development at the right timing.

This paper makes several contributions. First, we theoretically deduce an exit mechanism for luxury tax.[3] Second, we derive the optimal tax rate for luxuries that flexibly depends on economic performance. Third and finally, our extended tax smoothing model implies that the government should pay temporary expenditures from original budgets (even though staying in a budget deficit), instead of levying luxury goods to avoid distorting social costs, unless it is indeed necessary due to some considerations (e.g., depressing overheated housing markets).

The rest of the paper is organized as follows. Section 1 starts with some important luxury consumption and tax issues. Section 2 describes the literature related to luxury tax. We set up a Ramsey-type model of the optimal consumption growth model with tax policy and describe tax smoothing in Section 3. Section 4 studies the equilibrium and the optimal tax rate. In addition, we use numerical simulation to discuss tax distortion. The final section provides a conclusion and offers some suggestions.

2. Related Literature

Tax reforms and luxury taxes are quite important to the government due to their multiple impacts on economic development, social stability, and so on. Hong Kong and Singapore, for example, introduced new types of stamp duties on residential property transactions in order to prevent their land and house prices from bubbles.[4] In October 2011, Hong Kong imposed an additional 15% stamp duty on property purchased by foreign buyers. In Singapore, buyers who are non-resident people and/or firms need to pay 10% more of a home's value due to the Additional Buyer's Stamp Duty (ABSD).[5] The first consideration of imposing sales tax is to avoid the over-consumption for goods. For example, imposing a special sales tax can increase the cost of goods effectively and reduce the individual consumption of those goods. The taxation of luxury goods may increase the consumption of normal goods, but it possibly decreases the consumption of luxury goods. According to Frank (1985), positional goods are defined as those commodities whose value depends relatively strongly on interpersonal comparison. In contrast, goods are called nonpositional goods when they do not depend on interpersonal comparison. Frank (1985) pointed out that luxury goods are one kind of positional goods. Under the condition of incomplete markets in the U.S., Mathieu-Bolh (2010) found that progressive taxation generates more social welfare than uniform consumption taxation does in the long run (i.e., 12.06% vs. 9.88%) and during the terms from the transition period to the steady state period (i.e., 12.45% vs. 10.52%) while progressive consumption taxation is imposed on both necessities and luxuries. Bagwell & Bernheim (1996) proposed that luxury goods have the Veblen effect for those consumers who are willing to buy high-priced luxury brands to appear their higher social status. Some people - who consume conspicuous commodities in an attempt to achieve higher levels of status than their true status levels - induce inefficiency in social welfare. Therefore, Ireland (1994) had mentioned the welfare loss involved and pointed out that a corrective tax could yield a Pareto improvement.

Nevertheless, taxation can lead to excess burdens and directly affect a country's economic development. A luxury tax may lead to larger welfare losses than its actual gains, as the experience in the U.S., so that normal economic behaviors may be distorted. Therefore, policymakers should be careful in enacting tax policy, especially for luxury tax.[6] Barro (1990) considered the externality of government expenditures financed by a flat-rate income tax on the private sector, and he demonstrated that taxation had a negative effect on economic growth and distorted economic activities. Mendoza et al. (1997), in analyzing 18 Organisation for Economic Co-operation and Development (OECD) countries from 1965 to 1991, found that the excise tax had a positive effect on private investments, whereas both payroll tax and capital gains tax raised negative impacts on private investments. Policymakers need to consider the advantages and disadvantages emerged from a new tax policy simultaneously - that is, an optimal tax rate. The complexity of luxury tax particularly needs to be clarified.

3. Theoretical Model

This paper uses an optimum intertemporal consumption dynamic model and the extended tax smoothing model to analyze the impact of luxury tax on economic behavior. To the best of our knowledge, this is the first study to incorporate the role of a social planner into a centralized economy. The deductions of our theoretical model are as what follows.

3.1 The consumer

Suppose that $\boldsymbol{x}$ indicates necessity goods (e.g., food), which are necessary no matter if personal income is low or high. When people's income or wealth reaches to a sufficient level, their additional disposable income is capable of purchasing luxury goods y (e.g., jewelry, sports cars, or deluxe mansions). Undoubtedly, the rich can be easier to afford luxuries than the poor. The group of wealthier consumers have a higher absolute value of total EIS (elasticity of intertemporal substitution, see Browning & Crossley, 2000). Luxury goods can be defined via their characteristics and prices. As a result, we propose that consumers will choose luxury goods (y) when personal income has exceeded a certain threshold (x_0). Assume that household i's utility function is expressed as

$$u(x,y)=\left\{\begin{array}{c} x \text{ when } x<x_0 \\ x_0+(x-x_0)y \text{ when } x>x_0, x_0\in\Re_{++} \end{array}\right\} \quad \text{......} (1)$$

where $\boldsymbol{x}_0$ is a constant. In an economy with status-seeking behavior, luxury goods also have high intertemporal substitution elasticities. Figure 1 illustrates a hyperbolic curve relationship between x and y. An increase in the price of y leads to a decline in the consumption of y. This thereby implies a decrease in wealth, and the *IC* curve moves downward to the new *IC* curve. The original equilibrium point E_0 also moves to the new equilibrium point E_1, in which the consumption of necessity goods x is raised, and the consumption of luxury goods y is reduced.

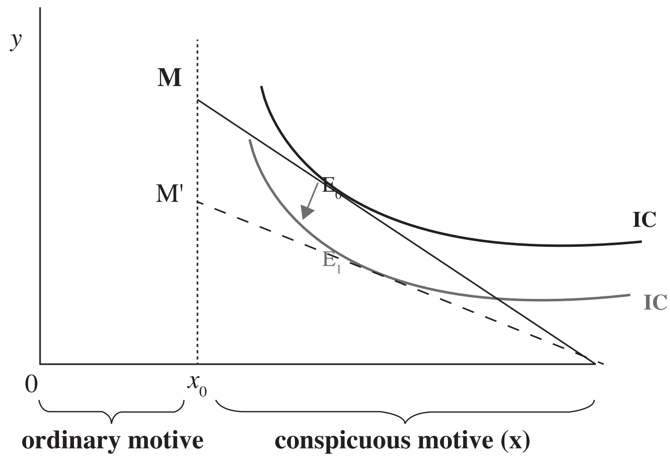

Figure 1. IC curve for x and y goods

Proposition 1.
If preferences are represented by a utility function that satisfies additivity over time and goods, then x and y can be regarded as the necessity goods and luxury goods, respectively, while the utility function in equation (1) meets a decentralized economy.
Proof: see Appendix A.1.

3.2 The producer

Assume that firms produce only two goods, including normal goods x and luxury goods y. The relevant consumption information of the goods x includes its nominal price and quality, and the quantity consumed is observable in public, whereas the consumption information of the goods y is observed only by a part of specified private consumers. In addition, all firms produce under the same production technology and tend to maximize profits, and the market is assumed to be completely competitive. Suppose that the representative household is a household-producer with k units of capital goods at time t, and the unit of output can be obtained from $f(k_t)$ with a luxury tax rate τ_y so that after-tax output at time t is $(1-\tau_y)f(k_t)$. The production function is defined as $Q = A\bar{L}f(k_t)$,[7] where A denotes the effectiveness of labor, and $\bar{L}$ is the total amount of labor employed. Combining with equation (A1.1), the goods and consumption markets to clear of the post-tax competitive equilibrium become

$$f(k_\tau^*) = p_x x_0 + p_x(x - x_0) + (1-\tau_y)p_y y \quad (2)$$

which is the production allocated between basic goods $p_x x_0 + p_x(x - x_0)$ and the post-tax consumption of luxury goods $(1-\tau_y)p_y y$ Note that the definite expressions of capital in several situations are as that: k denotes capital stock; k_τ denotes after-tax of capital; k^* is capital at steady-state, and k^*_τ indicates the post-tax capital at market equilibrium. Therefore, we express the household-representative net output after tax from equation (2) and then differentiate it with respect to y. It yields that

$$\frac{\partial f(k_{\tau}^{*})}{\partial y}=(1-\tau_{y})p_{y} \qquad (3)$$

3.3 Households

We further denote the variables r as interest rate, ρ as a discount rate, i.e. rate of time preference, θ as the elasticity of substitution for consumers, g as the economic growth rate, n as growth rate of population, and L(t) as the total population of the economy - that is, $L(t) = e^{nt}$ Given technology growing at rate g, we have $A(t) = e^{gt}$. Moreover, we consider the Ramsey-Cass-Koopmans framework with tax policy,[8] and $f(k)$ indicates the production output. We assume a two-goods production technology – that is, the output contains only two kinds of goods, and it is thereby the same for consumption markets. We define $c(t)$ as consumption per unit of effective labor. Thus, $C(t) = A(t)c(t)$ indicates consumption per worker, where $A(t)$ is the effectiveness of labor at time t. We assume that the economy is populated by a large number of identical and infinitely lived individuals by a perfect foresight. A representative household[9] pursues and maximizes the objective function utility expressed as follows:

$$U=\int_{t=0}^{\infty}e^{-(\rho-n)t}u(C(t)\ )dt\ ,where\ u(C(t))=\frac{C(t)^{1-\theta}}{1-\theta}\ ^{10} \qquad (4)$$

subject to

$$\dot{k}=f(k)-c-(n+g)k \qquad (5)$$

Let τ_y be the tax rates on luxury consumption, and D is the fixed proportion for its tax rates. In addition, the government taxes the luxury goods, transfers those taxed revenues to households, maintains a balanced budget at all times, and rebating all tax revenues as lump-sum transfers (TR):

$$TR=\tau_{y}f(k)+\tau_{y}c_{y} \qquad (5.1)$$

where c_x and c_y are the consumption of the necessity and the luxury goods, respectively. Moreover, we let $c_x + c_y = c$. Herein we might interpret c_x as "necessary" or priority goods consumption and c_y as "luxury" goods consumption, which are purchased only for acquiring sufficient necessities as the expression in equation (1). Note that this expression is also similar to Bagwell & Bernheim (1992). The capital accumulation equation given by (5) is rewritten as:

$$\dot{k}=(1-\tau_{y})f(k)-(1+\tau_{y})c-(n+g)k+TR \qquad (5.2)$$

The arrangement is as follows:

$$\dot{k} = (1-\tau_y)f(k) - (1+\tau_y)c - (n+g)k + \tau_y f(k) + \tau_y c_y \quad (5.3)$$

$$\dot{k} = f(k) - \left(1 + \underbrace{(1 - c_y / c)}_{D}\tau_y\right)c - (n+g)k \quad (5.4)$$

The economy-wide resource constraint yields:

$$\dot{k} = f(k) - (1 + D\tau_y)c - (n+g)k \quad (5.5)$$

Thus, we have

$$\text{H(k,c,}\lambda\text{)} = u(C)e^{nt} + \lambda\left[f(k) - (1 + D\tau_y)c - (n+g)k\right] \quad (6)$$

or the substitute at time t

$$\text{H(k(t),c(t),}\lambda\text{(t))} = u(c(t)A(t))e^{nt} + \lambda(t)\left[f(k_t) - (1 + D\tau_y)c(t) - (n+g)k(t)\right] \quad (7)$$

where $\lambda(t)$ is the co-state variable associated with the budget constraint and the shadow value of the private capital stock at time t. Generally speaking, $\lambda(t)$ can be interpreted as a shadow price, indicating an increase in the value function (i.e., social welfare) for a small (marginal) increase in $u(C(t))$. In other words, $\lambda(t)$ gives the social value of individual utility, implying a marginal social cost of public funds (MCF) due to taxation imposition. The MCF is a measure of the cost of raising tax revenue from a particular tax instrument. The MCF can be used to identify the changes of tax structure that raise welfare amounts while keeping expenditures constant.

3.4 The social planner's solution

This subsection considers the problem of social planners who choose the optimal path and take into account the tax impact of luxury consumption on economic performance. To solve the dynamic optimization problem, we use a current-value Hamiltonian function to derive the following conditions:

$$\frac{\partial H}{\partial c} = 0 \Rightarrow (1 + D\tau_y)\lambda = u'(cA)e^{nt}A = u'(cA)e^{(n+g)t} \quad (8)$$

After taking the logarithm differential, we get

$$\ln(1 + D\tau_y)\lambda = \ln u'(cA) + (n+g)t$$

and

Therefore,

$$\frac{\dot{\lambda}}{\lambda} = \left(\frac{u''(C)C}{u'(C)}\right)\frac{\dot{C}}{C} + (n+g) = \frac{1}{\theta}\left(\frac{\dot{c}}{c} + g\right) + (n+g)\,.$$ [11]

$$\frac{\dot{c}}{c}=\frac{1}{\theta}\left[-\frac{\dot{\lambda}}{\lambda}+n+(1-\theta)g\right] \quad (9)$$

Moreover,

$$\frac{\partial H}{\partial k}=-\dot{\lambda}+\rho\lambda \quad (10)$$

$$\frac{\partial H(k(t),c(t),\lambda(t))}{\partial k}=\lambda(r-n-g) \quad (11)$$

The equivalent conditions of equations (10) and (11) thereby yield

$$-\frac{\dot{\lambda}}{\lambda}=r-(n+g+\rho) \quad (12)$$

Put equation (12) into (8), it yields that

$$\frac{\dot{c}}{c}=\frac{1}{\theta}\left[-\frac{\dot{\lambda}}{\lambda}+n+(1-\theta)g\right].$$

The optimal growth path is given from the usual Euler relationship:

$$\frac{\dot{c}}{c}=\frac{r-\rho-\theta g}{\theta} \quad (13)$$

Equation (13) is also known as the Keynes-Ramsey equation, indicating that along with the optimal consumption path, the growth rate of consumption is positive whenever the net interest rate ($r–\theta g$) exceeds the time preference rate (ρ). Moreover, assuming $r = f'(k)$ and incorporating equation (3) into Euler equation (13), the non-anticipatory tax policy, with respect to the maximum utility of the effective consumption growth rate per unit for each house, is

$$\frac{\dot{c}}{c}=\frac{(1-\tau_y)p_y-\rho-\theta g}{\theta} \quad (14)$$

Equation (14) indicates the long-term influence of taxes on economic growth. The direct effect of higher taxes on luxury goods reduces the net interest rate r for a given ratio of physical to human capital. In the balanced growth path $\dot{c}=0$, equation (14) can be simplified as

$$(1-\tau_y)p_y=\rho+\theta g \quad (15)$$

In equation (15), the total differential formula is

$$dp_y - d\tau_y \cdot p_y - \tau_y dp_y = d\rho + \theta dg + gd\theta \quad (16)$$

After arrangement, we have

$$\tau_y = 1 - \left(\frac{p_y d\tau_y + (d\rho + \theta dg + gd\theta)}{dp_y} \right) \quad (17)$$

Suppose that $\frac{p_y d\tau_y + (d\rho + \theta dg + gd\theta)}{dp_y} = \eta$, equation (17) can be replaced by

$$\tau_y = 1 - \eta \quad (18)$$

The equilibrium (or the optimal) luxury tax rate can be expressed as the simple form τ^*_y=1-η, where η consists of several economic variables: ρ, θ, g, and p_y. If there were no consumption externality consideration and the other conditions remained, then an imposed luxury tax might improve economic growth rate (g).

Furthermore, we are concerned about the relationship among luxury tax, technological progress rate, and capital stock. First, we assume that ρ and θ are constant and are given exogenously so that $d\rho=d\theta=0$. Then, equation (16) becomes

$$dp_y - d\tau_y p_y - \tau_y dp_y = \theta dg \quad (19)$$

and

$$\frac{dg}{d\tau} = \frac{-p_y}{\theta} < 0 \text{ , where } p_y>0 \quad (20)$$

When social status is determined by its corresponding wealth, the neutrality of constant consumption taxation does not hold anymore. Consumption tax can impact an economy's overall consumption/capital ratio negatively, and it thereby positively affects its steady-state growth rate.

Remark 1.

Taxation on luxury goods from no initial distortion can decrease the steady-state capital stock.

Luxury tax raises a negative effect on the relationship between the economic growth rate or the rates of technological progress and capital. Remark 1 raises an important implication for policy that taxing luxury goods may harm capital accumulation, as found in Ikeda (2006).

Second, if ρ, θ, and g are constant and are given exogenously, then $d\rho=d\theta=dg=0$. Equation (20) can be substituted with $dp_y - d\tau_y p_y - \tau_y dp_y = 0$. It implies that

$$(1 - \tau_y) dp_y = p_y d\tau_y \quad (21)$$

and

$$\frac{dp_y}{d\tau_y} = \frac{p_y}{1-\tau_y} > 0 \quad (22)$$

where τ_y is proportional to p_y, which indicates the characteristics of ad valorem. For a given (positive) exogenous variable B_y (taxable base) and taxation T, we can differentiate the tax rate formulas $\frac{T}{B_y} = \tau_y$ and get that

$$dT = B_y d\tau_y \quad (23)$$

According to the results of equations (22)-(23), we obtain the following proposition.

Proposition 2.

When a luxury commodity price is high, taxation (T) has an increasing trend.

Proposition 2 spells out that the more a luxury goods price grows, the higher luxury tax rate the government should charge. As argued in Ng (1987), an increase in luxury commodity prices will not affect consumers' utility, and the government thereby should impose a higher tax on luxury goods.

We further assume that the government charges luxury goods with τ tax rate. For a given real interest rate $r(t)=(1-\tau)f'[k(t)]$,[12] the growth rate of consumption $(\dot{c}/c)$ has the following deduction and implication:

$$\frac{\dot{c}}{c} = \frac{r-\rho-\theta g}{\theta} \Rightarrow \frac{\dot{c}}{c} + g = \frac{r-\rho}{\theta} \Rightarrow \theta = \frac{r-\rho}{(\dot{c}/c)+g}.$$

Therefore, the intertemporal elasticity of substitution is less than one, and the case can deal with capital or goods taxation.[13] In following, we will discuss the condition with respect to the variable θ.

Assumption A:

For the given condition $\theta = \frac{r-\rho}{(\dot{c}/c)+g} > 1$, we deduce that

$$\tau < 1 - \frac{1}{r}\left(\frac{\dot{c}}{c} + g + \rho\right) \quad (24)$$

where $\frac{1}{r}\left(\frac{\dot{c}}{c} + g + \rho\right) > 0$. This result conflicts with the proposition in Rauscher (1997), in which a tax should not be imposed if $\theta > 1$, or the tax rate τ is inconsistent with a rationale value.

Assumption B:

Under the condition $\theta = \frac{r-\rho}{(\dot{c}/c)+g} < 1$, we have that

$$r-\rho<\frac{\dot{c}}{c}+g \Rightarrow r-\rho-g<\frac{\dot{c}}{c} \Rightarrow \frac{\dot{c}}{c}>r-\rho-g \quad (25)$$

In equation (25), the optimal path is greater than r–ρ–g. The process of imposing luxury goods at $\tau\%$ tax rate is deduced as follows:

$$\theta=\frac{r-\rho}{(\dot{c}/c)+g} \Rightarrow \theta=\frac{(1-\tau)f(k)'-\rho}{(\dot{c}/c)+g}<1$$

Hence we get

$$1-\frac{1}{r}(\frac{\dot{c}}{c}+g+\rho)<\tau \quad (26)$$

Therefore, we have the following proposition.

Proposition 3.

The government should start the exit mechanism of the luxury tax while it satisfies the following condition:

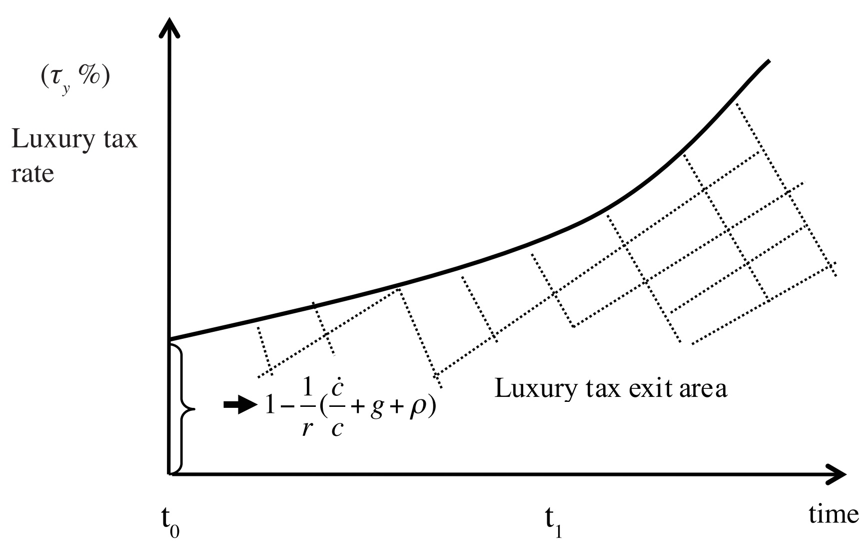

Figure 2. Diagram of the relationship between luxury tax and time

$$\tau < 1 - \frac{1}{r}(\frac{\dot{c}}{c} + g + \rho) \cdot$$

Figure 2 illustrates the relationship between luxury tax (τ_y) and time (t). The exit mechanism of the luxury tax generates an exit region that varies with time t. If authorities continue to levy within tax exit area, then it may bring out extra costs of tax collection. Moreover, the country can suffer more excess burdens, and the economy can also experience the dynamic inefficiency problem.

3.5 Government

Assume that the government can finance public expenditures G(t) through executing a luxury tax with an average rate τ_y. Suppose that the government imposes luxury tax on only two kinds of luxury goods. One kind of luxury goods has high price p_{y1} with quantity y_1, and the other has low prices p_{y2} with quantity y_2. The no-borrowing constraint is binding for both goods. Hence, one part of government expenditures and capital stems from the luxury taxes, which satisfies

$$G_1=\tau_y p_{y1} y_1 \quad (27)$$

and

$$G_2=\tau_y p_{y2} y_2 \quad (28)$$

When government spending exceeds tax revenue, the government runs a budget deficit (i.e., $G=G_1$); when $G=G_2$, the government has a budget surplus. The level of taxes is determined by the intertemporal budget constraint which implies that the present value of spending (which is exogenously given) has to be equal to the present value of taxes. Therefore, budget deficits and surpluses are used as a buffer; deficits occur when spending is temporarily high, and surpluses occur when it is low. A high tax rate causes a cost on society by discouraging economic activity. Because this disincentive is so costly at particularly high tax rates, the total social cost of taxes is minimized by keeping tax rates relatively stable, rather than making them high in some years and low in others. A tax-smoothing policy keeps tax rates smooth; a deficit is necessary in years of unusually low revenue or unusually high expenditure.

3.6 Extension of Barro's tax smoothing model

Herein, we extend Barro's (1990) tax smoothing model to examine government expenditures associated with luxury tax rates. Assume that both output and the real interest rate r are constant, the level of outstanding government debt keeps steady, and distortion costs are quadratic. Suppose two possible values with respect to government expenditures G_1 and G_2 (see equations (27) and (28)) with $G_1>G_2$. To facilitate the derivation operation, the $\tau_y p_{y1} y_1$ and $\tau_y p_{y2} y_2$ are shown as following G_1 and G_2, respectively. The transitions between the two values obey the Poisson process. If in the case of $G=G_1$,

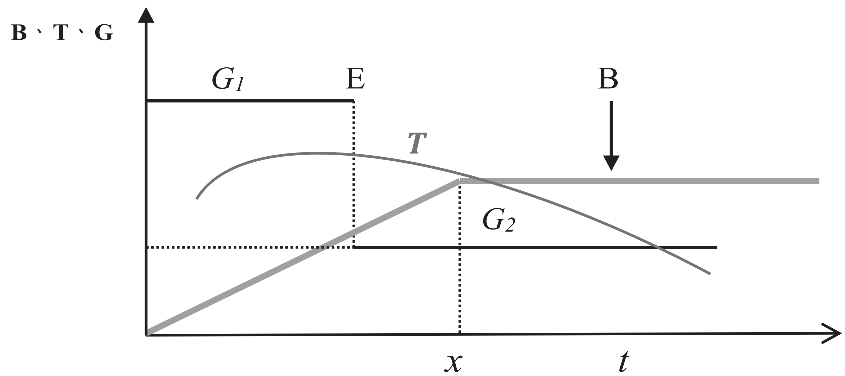

Figure 3. Diagram of government spending (G), budget (B), luxury tax (T), and time (t)

the probability of falling to G_2 to purchase per unit time is α. If G=G_2, then the probability of raising to G_1 for purchases per unit time is β. Figure 3 illustrates the conditions. When the government's expenditures grow from levying luxury tax and achieve a high level T_1, we have that

$$\int_{t=0}^{\infty} e^{-rt} T_1 \, dt = P_1 \tag{29}$$

Solving the integral with the substitution of $\frac{1}{r}T_1 = P_1$, we get

$$P_1 = \frac{\alpha G_2 + (\beta + r)G_1}{r(\alpha + \beta + r)} + B \tag{30}$$

Proof: see Appendix A3.

Now, if we substitute $\frac{1}{r}T_1 = P_1$ into equation (30), we have

$$T_1 = \frac{\alpha G_2 + (\beta + r)G_1}{(\alpha + \beta + r)} + rB \tag{31}$$

Similarly, when G=G_2, the government should impose luxury tax T_2, which can be expressed as

$$T_2 = \frac{(\alpha + r)G_2 + \beta G_1}{(\alpha + \beta + r)} + rB \tag{32}$$

Figure 3 illustrates the path of government spending (G), budget (B), and luxury tax (T) with time (t) that varies according to equations (31) and (32).

From Figure 3, we can see that the path of tax revenue is decreasing during an interval with a constant G conditional on budget B. In general, we propose that the changes of budget deficit have the form:

$$\dot{B} = G - T + rB \tag{33}$$

where the sign of a dot added on variables represents the time derivative; for example, $\dot{B}=\left(\frac{dB}{dt}\right)$. According to equation (33), the tax path can be expressed as

$$\dot{T}_1 = r\dot{B} = r(G_1 - T_1 + rB)\ .$$

To replace equation (31) with the above equation, we can obtain that

$$\dot{T}_1 = r\left[G_1 - (\frac{\alpha G_2 + (\beta + r)G_1}{\alpha+\beta+r}) + rB\right].$$

Furthermore, we rearrange it to be:

$$\dot{T}_1 = r\left[\ \frac{(\alpha+\beta+r)G_1 - \alpha G_2 - (\beta+r)G_2}{\alpha+\beta+r}\ \right] = \frac{\alpha r(G_1 - G_2)}{\alpha+\beta+r} > 0 \quad \text{(34)}$$

In equation (34), the case of $G = G_1$ indicates that the government runs a budget deficit since government expenditures are over budget, and we can expect the situation of risen luxury tax in the future, owing to the increasing interest rates on the extra debt. In addition, when G falls to G_2, the tax revenue will go from T_1 down to T_2. The path of tax revenue under the condition $G=G_2$ is again driven by the path of the budget deficit so that the change of tax revenue in period 2 can be expressed as

$$\dot{T}_2 = \dot{B} = r(G_2 - T_2 + rB)\ .$$

Incorporating this into equation (30), it yields that

$$\dot{T}_2 = r\left[G_2 - (\frac{\beta G_1 + (\alpha+r)G_2}{\alpha+\beta+r} + rB) + rB\right] = r\left[\ \frac{(\alpha+\beta+r)G_2 - \beta G_1 - (\alpha+r)G_2}{\alpha+\beta+r}\ \right]$$

$$= \frac{\beta r(G_2 - G_1)}{\alpha+\beta+r} < 0 \quad \text{(35)}$$

In equation (35), when G equals to G_2, the government stores budget revenue since the government's budget is higher than its expenditure. The luxury tax rate will thereby decrease with a span of time in the future, owing to decreasing interest rates on the budget deficit. Intuitively, if the government realizes that there will be a high probability of increasing its expenditures in the future, the government will keep its budget revenue to smooth its expenditures in the future. As a result, we obtain the following proposition.

Proposition 4.

The results from equations (34) and (35) plotted in Figure 3 show that if government spending G rises to G_1, then luxury tax will rise to T_1 with time; at the moment that government spending G falls to G_2, then the tax revenue will drop from T_1 to T_2.

According to the derived results above, we have several important implications. First, if tax revenues are maintained smoothly and thereby there is an increase in the government's temporary expenditures, then the government should finance its budgets via other financial channels (e.g., issuing bonds), rather than by levying luxury tax. These issued bonds can be redeemed when the economic situation gets better. Luxury tax is not an appropriate policy for short-term economic development due to the excess burdens of tax collection. Therefore, financing by issuing bonds is a better policy than the policy of levying luxury tax for a government's temporary expenditures.

Second, the government pursues the minimization principle of distortions for the costs of tax collection or excess burden while searching for the optimal intertemporal tax structure, but the budget deficit helps to maintain the efficiency of taxes.[14] When the government adopts imposition taxation to minimize the excess burden of taxation principles, the tax rate will be maintained at a constant level so as to ensure a balanced budget, and budget deficits appear when unanticipated expenses arise.

Third and finally, the optimal tax rate should be maintained at the level $\tau^* = 1 - \eta$. If the luxury tax rate is less than $1 - \frac{1}{r}(\frac{\dot{c}}{c} + g + \rho)$, dynamically inefficient will be generated.[15] Moreover, the target of injecting into public revenues cannot be practiced by imposing luxury tax on various conspicuous consumptions for the rich. Instead, low luxury taxes only drive increasing costs. As a result, a luxury tax should have an exit mechanism.

4. Simulation Analysis and Results

In this section, we simulate the case of luxury tax in Taiwan. In choosing the parameter values for the model, we employ the following strategy. First, we select a subset of more common parameters in accordance with similar work in this field or in the existing literature, such as Romer (2006), Barro & Sala-i Martin (2004), and Yang et al. (2011). Then, the remaining parameters are set so as to match the various features of Taiwan's data. To obtain the optimum luxury tax rate for Taiwan, as expressed in equation (17), we incorporate a discount rate (ρ), the elasticity of substitution (θ), an economic growth rate (g), capital accumulation (k), and other economic variables. Note that the discount rate (ρ) is traditionally set as ρ=0.02, and the economic growth rate (g), following Romer (2006), is set at 3%. We collect the other variables or proxy variables from several databases described in Appendix B (Table B1). The empirical data we apply cover the period from January 2012 to May 2013 in Taiwan.

4.1 Calibration

Under some specified functions, we set the basic parameters as follows:

Taxes with variation parameters: dT=0.0421, and dp_y=0.05. Note that the value dT herein is regarded as the proxy of $d\tau_y$.[16]

Taste parameters: g=0.03, r=0.035, ρ=0.02, θ is set as 0.1, 0.3, 0.5, 0.7, and 0.9 respectively, $d\rho$=0, $d\theta$=0.2 and dg is in turn set as 0.01, 0.02, 0.03, 0.04, and 0.05.

Relative price: the threshold of levying luxury tax is set to be p_y=103,000 (USD), which is the threshold of levying luxury tax for a deluxe car in Taiwan.

4.2 Simulation results

Figure 4 shows the trends of total luxury tax revenue and the variation during sample periods, in which the tax revenue reaches to a peak in March 2012. Furthermore, Figure 5 illustrates the simulation results of the model,[17] aiming at the luxury tax exit mechanism with a particular economic

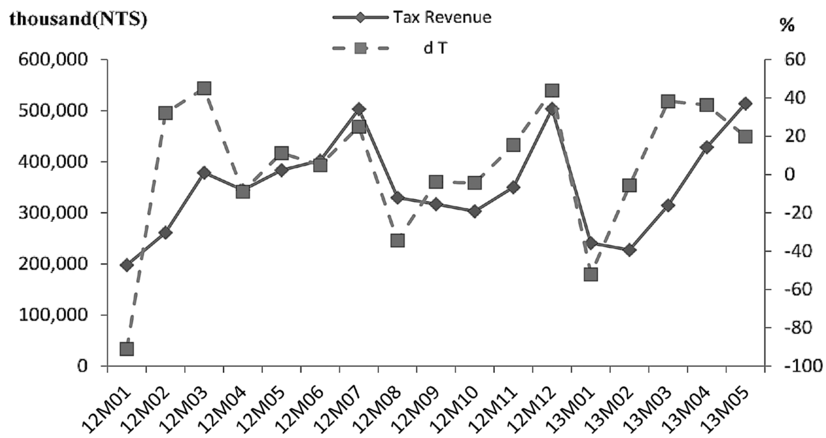

Figure 4. The trends of tax revenue and its variation during sample periods

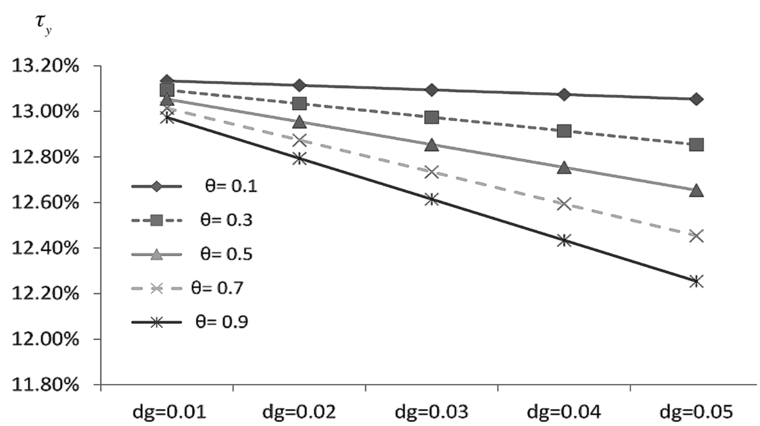

Figure 5. Optimum luxury tax rate against variation growth rate

growth rate of the optimal tax rate. The optimum luxury tax we simulate is approximately 13%. Even with different levels of θs, the optimal tax rate is still estimated in the range between 12% and 13%. In particular, Figure 5 also illustrates that the optimal tax rate is decreasing with the intertemporal elasticity of substitution θ, which is consistent with the findings in Mandel (2009).

4.3 Optimal taxation policy

The result displays the effect of different economic performances on the optimal tax rate of luxuries. Accordingly, the optimal tax rate region against economic growth, as well as Figure 6 below, illustrates that the curve shown as steady state (balanced growth path) may be in or close to the prohibitive range of the Laffer Curve.[18]

The previous literature tends to calculate the average effective tax rate rather than the optimal tax rate because the optimal tax rate is always difficult to find out, and it depends on various economic conditions. The optimal tax rate in the past may not be applied appropriately nowadays.

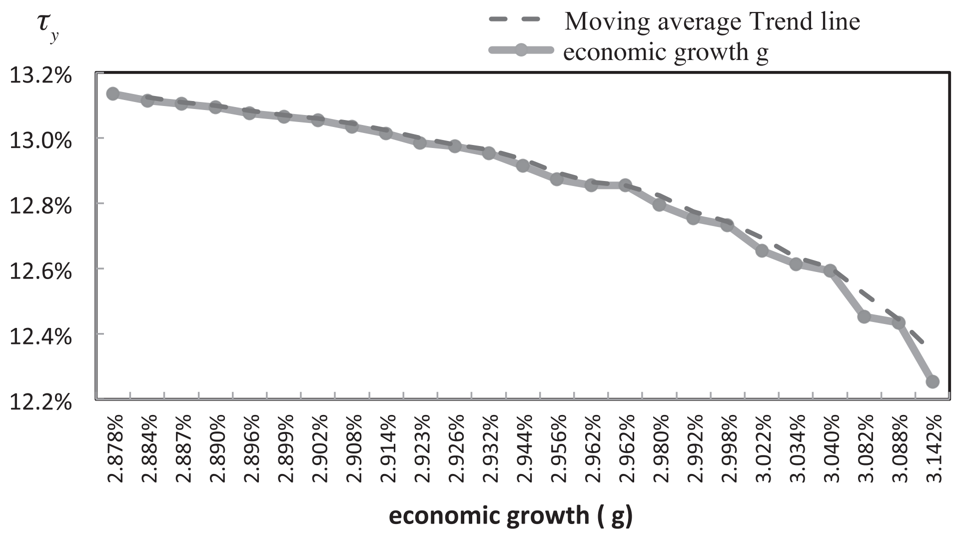

Figure 6. Optimal tax rate region against economic growth

Cary & Harry (2000) calculated, aiming at the OECD countries from 1965 to 1991, and found that the average effective tax rates in the subperiods were 16.1% during 1980-85, 17.2% during 1986-90, and 17.1% during 1991-97. These empirical results are somewhat different from some previous findings. For example, Mendoza et al. (1994) employed the U.S. data from 1965 to 1988 and found the capital tax rate at about 0.43, consumption tax rate at 0.06, and payroll tax rate at about 0.25. The simulated optimal tax rate in our study is closer to Taiwan's practical luxury tax rate. Concerning the formulation of the optimal tax rate in this paper, the performance of numerical simulation includes not only a framework, but also the robustness with respect to feasibility approaches. The result is slightly different from the findings of Mathieu-Bolh (2010) due to the different conditions of incomplete markets with progressive consumption taxation and the taxing of necessities and luxuries at different

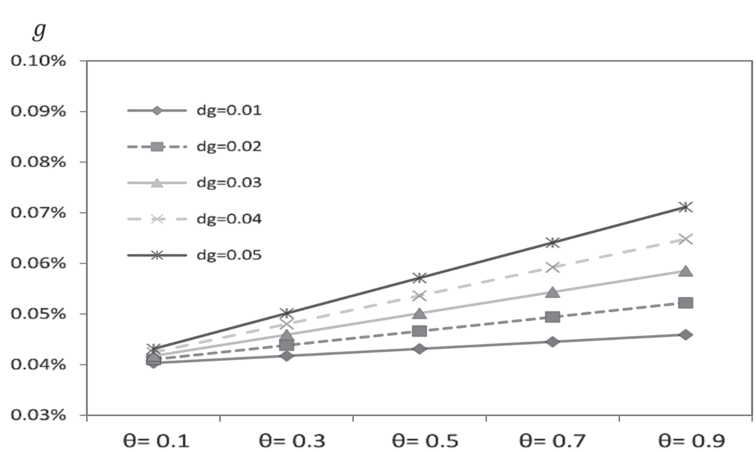

Figure 7. Economic growth rate against different θ in optimum tax rate region

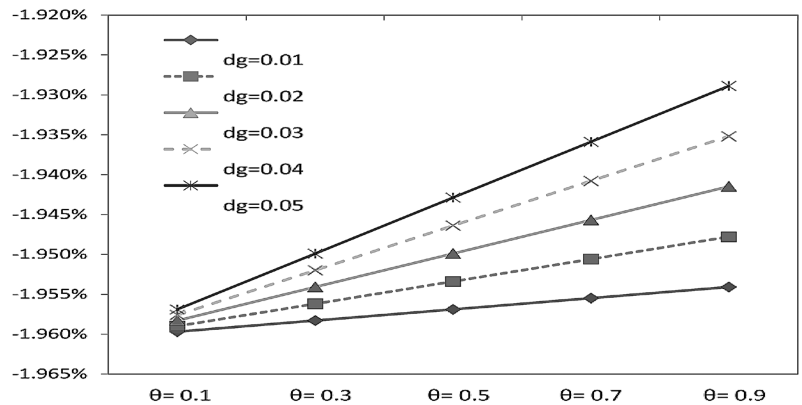

Figure 8. Growth rate of per capita consumption against different θ in optimum tax rate region

rates compared to Mathieu-Bolh's study. When the economic growth rate is below 0.07% (or 0% in the GDP growth rate per year), the government needs to think about the exit mechanism for luxury tax as shown in Figure 7. On the other hand, Figure 8 shows the starting point of the exit mechanism when the average growth rate of consumption is less than -1.925%.

The authorities should think about luxury tax determination - that is, starting the exit mechanism of luxury tax. In the U.S., the economic growth rate fell down to -0.2% in 1991. The luxury tax was implemented, and the initial tax revenue was expected to collect $9 billion at that time. Unfortunately, this goal failed and the manufacturing of luxury commodities suffered huge losses instead, which resulted in a worse economic situation. As a consequence, the luxury tax was replaced in 1993 and the exit mechanism began. Similarly, there have been some debates on the issue of luxury taxes in Taiwan recently. Taiwan's authorities have attempted to prevent a housing bubble from occurring due to speculators in future housing markets. In addition, levying luxury taxes further reduces the inequality between rich and poor people to perform social justice. However, this is just a conceptual problem. The excess burdens that result from luxury taxes discourage a country's economic development. Importantly, the exit mechanism should be done according to the rules of the country's economic growth data. The timing of the exit mechanism should be avoided implement as changes in tax structures are often tied to who is in power and their political leanings. It therefore follows that a policy implemented according to a rule will achieve lower distortionary taxation than a discretionary policy.

5. Conclusions and Implications

Jean Baptiste Colbert said: "The art of taxation consists in so plucking the goose as to obtain the largest possible amount of feathers with the smallest possible amount of hissing."

This paper focuses on the study of an optimal luxury tax and its timing of starting an exit mechanism. Several findings and suggestions are offered as follows. First, the paper deduces that the optimal luxury tax rate is $\tau^{*}=1-\eta$, where economic growth rates, or the rates of technological progress

g, are the determinants of η. Moreover, as learned from Remark 1, the luxury tax is only imposed appropriately while the economic growth rate is at a general level. The wrong timing of implementing a luxury tax would lead to large losses. The unsuccessful experience in the U.S. in 1990 is a mirror (see, for example, Rosen & Gayer, 2008).

Second, according to our theoretical demonstration, a government should start the luxury tax exit mechanism when the optimal tax rate is lower than a specific equilibrium value we estimated. If this does not happen, the country will suffer more excess burdens, and the economy will experience the dynamic inefficiency problem. As found in the theoretical framework, there will be a lock-in effect for taxation on capital gains when taxpayers consider the cost of taxes on the sale of assets or securities, which makes taxpayers unwilling to change their asset portfolios. In particular, luxury tax is generally higher than other kinds of taxes. Policymakers must be careful in manipulating such a tax to prevent investors from the lock-in effect and thereby a bias in capital allocation and depressed market activities.

Third and finally, this paper simulates the case in Taiwan and offers a practicable exit mechanism. For example, the authorities should execute the exit mechanism when the annual economic growth rate is less than 0.07% and the average growth rate of consumption is less than -1.9%. The optimal growth rate can be attained in a decentralized economy only if the consumption tax rate is time-varying (see, for example, Rauscher, 1997, and Fisher & Franz, 2000, for other models about economic growth). An increasing (decreasing) tax rate over time leads to an increase (decrease) in future commodity prices.

The advantages and disadvantages of the luxury tax issue have been discussed since Taiwanese tax authorities levied an additional tax based on the Specifically Selected Goods and Services Tax Act. Here we offer the optimal luxury tax and an appropriate exit mechanism that depends on one country's macroeconomic conditions, such as economic growth, technological progress rates, and so on. Our findings involve several policy implications and are beneficial for policymakers. For example, luxury tax is the tool considered by Taiwan's government recently to restrain speculative activities of housing markets, for which Proposition 2 is available to mitigate soaring housing prices in Taiwan. From Proposition 3, our theoretical demonstration offer an appropriate tax exit area, by which it reduces possible extra costs of tax collection and excess burdens. In addition, Proposition 4 implies the co-movement of a government's spending and luxury tax that implicates the optimal "dynamic" luxury tax rate. Overall, we believe that this paper is referable and valuable to luxury tax policy.

Note

1: According to the Ramsey Rule, the tax rate should be inversely proportional to commodity price elasticity so that the excess burdens resulting from taxing are relatively less.

2: In addition to luxury tax regarded as one kind of fiscal policies, monetary policy can also impact housing markets through the channel of interest rate.

3: Herein, the exit mechanism for luxury tax means that the government should defer or stop levying luxury taxes. In practice, the government may set the thresholds of executing and stopping luxury taxes based on our suggestions in this paper.

4: There are also several factors that affect housing markets (see, for example, Chen et al., 2011; Zeng et al., 2013). On the other hand, in addition to fiscal policies (e.g., luxury tax), monetary policies are also influential to housing markets (e.g., Chen et al., 2012), the banking sector (e.g., Chen et al., 2014), etc.

5: Source: *South China Morning Post*, Hong Kong, October 28, 2012. http://www.scmp.com/news/hong-kong/article/1071453/new-15-cent-stamp-duty-may-hit-expatriate-homebuyers.

6: Ohanian (1997), who analyzed Britain's tax policy during World War II and the postwar period, found that the British government substantially increased the income tax rate to raise funds during World War II but did not decrease the tax rate after World War II. As a result, there was a huge tax distortion in the British economy, which caused a slowly developing economy for a long time.

7: Chen and Hsu (2009) argued that a single good is produced by $Q=f(k)$, where Q is output per capita and k is capital stock per capita with an initial value $k(0)$, and capital stock does not depreciate. The A, L production factors are given. The technology exhibits standard properties in a neoclassical production function, and the Inada conditions are expressed as $\frac{\partial f(k)}{\partial y}\Big|\dot{k}=0 \approx \frac{\partial f(k)}{\partial k}\Big|\dot{k}=0 \approx f'(k)=r$, because $f(.)$ is an increasing concave function of k. $f'(k)>0$, $f''(k)<0$, and $\lim_{k\to\infty} f(k)=\infty$.

8: We thank an anonymous referee for many helpful comments on a preliminary draft. In general, the consumption utility function $u(C(t)) \approx u(x,y)$, i.e., $u(C(t))$ is the implicit function of x and y. Because $u(C(t))$ consists of the consumption of x and y goods, optimal consumption has been solved in equation (A4.1), and optimal consumption (x & y) post-tax on luxury goods can also be solved, i.e., $\left(x_0+\frac{w-p_x x_0}{2p_x}, \frac{w-p_x x_0}{2p_y(1+\tau_y)}\right)$. Therefore it is no longer substituted into equation (1). Here, we consider the straightforward approach into the neoclassical growth model with a Cobb–Douglas production function to solve the steady-state values of the other economic variables

($\dot{c}/c, \tau_y, k, \theta, g$) by equations (4)-(13). Furthermore, $u(C(t)) = F(\tau_y, k, \theta, g)$, where $u(\bullet)$ is the function of the economic variables (i.e., τ_y, k, θ, and g) $u(C(t))$ denotes the implicit function of equation (1).

9: The authors would like to thank anonymous reviewers for their useful comments. Most of the consumption theory literature with respect to consumption growth are based on the *i.i.d.* assumption (see Abel, 2005), and we consider that consumers have the same utility function (e.g., Ljungqvist & Uhlig, 2000) and follow the existing research on the analysis of luxury tax impacts (e.g., Rauscher, 1997; Ikeda, 2006).

10: In particular, the utility function $u(\bullet)$ is of endogenous growth models. It is necessary to use a CRRA felicity in order to ensure the existence of a balanced growth path. Particularly, it is known that a CRRA felicity with individual and average consumption is homothetic along with the equilibrium path (Chen & Hsu, 2009).

11: More about $1/\theta$ see Appendix A2.

12: For more details, one can refer to Barro (1990).

13: Rauscher (1997) pointed out that capital must be taxed along with the optimal path if $\theta < 1$, and capital should be subsidized while $\theta > 1$. If the intertemporal elasticity of substitution is small, the competition of social status stimulates economic growth, and capital accumulation should be slowed down.

14: Efficiency is an important issue that has been discussed for a long time and over several fields, such as stock markets and energy markets (see, for example, Malkiel, 2003; Chen et al., 2014).

15: Phelps (1961) pointed out that if the luxury tax rate is lower at a particular level, a firm will overproduce luxury goods, and its capital stock will be over the optimal level of the golden rules. In this case, the economy is in a state of dynamic inefficiency; Romer (2006) mentioned this concept too.

16: Please see equation (23).

17: The authors thank Jim Alm for insightful suggestions. We use the actual consumption data of luxury goods to simulate a luxury tax rate that is not just over the level of aggregate consumption and illustrate luxury goods with y in the preceding subsection 3.1.

18: The Laffer Curve obviously illustrates the point at which taxes become so high that tax revenue begins to decline; otherwise it is known as the Prohibitive Range of the curve (see, for example, Laffer, 2004).

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Appendix

Appendix A1

A luxury good has an income elasticity superior to one and is purchased by people above a certain income level. Suppose $w>p_x x_0$ for a constant x_0 and a budget constraint on wealth w. We have

$$Max\ \ x_0+(x-x_0)y \quad \text{(A1)}$$

s.t.

$$p_x x_0+p_x(x-x_0)+p_y y=w \quad \text{(A1.1)}$$

Without a loss of generality, we let $z=x-x_0$. Then we can get

Max $Max\ z\bullet y\ \ s.t.\ \ p_x z+p_y y=w-p_x x_0=w_1$.

To solve it with the method of Lagrange multipliers, we have

$£=zy+(w_1-p_x z-p_y y)$

The $f.o.c.$ with respect to z, y, and λ, respectively, gives the following equations:

$$\frac{\partial £}{\partial z}: y-\lambda p_x=0 \quad \text{(A2)}$$

$$\frac{\partial £}{\partial y}: z-\lambda p_y=0 \quad \text{(A3)}$$

$$\frac{\partial £}{\partial \lambda}: w_1-p_x z-p_y y=0 \quad \text{(A4)}$$

In particular, we have $\frac{\partial y}{\partial z}=\frac{p_x}{p_y}$ from dividing (A2) by (A3). Substitute it into equation (A4), we get $z=\frac{w_1}{2p_x}$ and $y=\frac{w_1}{2p_x}$ with $z=x-x_0$ As a result, we have

$$x=x_0+\frac{w\ -p_x x_0}{2p_x} \text{ and } y=\frac{w\ -p_x x_0}{2p_y} \quad \text{(A4.1)}$$

We further solve the second-order condition for the constrained optimization. We now derive the bordered Hessian determinant of the Lagrange function as follows.

$$H^* = \begin{vmatrix} \frac{\partial^2 £}{\partial z \partial z} & \frac{\partial^2 £}{\partial z \partial y} & -p_x \\ \frac{\partial^2 £}{\partial y \partial z} & \frac{\partial^2 £}{\partial y \partial y} & -p_y \\ -p_x & -p_y & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & -p_x \\ 1 & 0 & -p_y \\ -p_x & -p_y & 0 \end{vmatrix} = 2p_x p_y > 0 \quad \text{(A5)}$$

where $H_1^* = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$ and it is thereby non-positive. Since $H^*_2 = H^* > 0$, this positive shows that the x and y values at $(x_0 + \frac{w - p_x x_0}{2p_x}, \frac{w - p_x x_0}{2p_y})$ have a satisfied maximum utility function with a strictly quasi-concave condition. Thus we are sure that the tangency point between the indifference cure and the budget line belongs to the local maximum points when the indifference cure is strictly convex to the origin. The *income elasticity of goods* for x and y goods are the income elasticity of demand with

$$\varepsilon_{xw} = \frac{d\ln w}{d\ln x} = \frac{dx}{dw}\frac{w}{x} \Rightarrow \frac{1}{2p_x}\frac{w}{x_0 + \frac{w - p_x x_0}{2p_x}} = \frac{w}{w + p_x x_0} < 1 \quad \text{(A6)}$$

and

$$\varepsilon_{yw} = \frac{d\ln w}{d\ln y} = \frac{dy}{dw}\frac{w}{y} \Rightarrow \frac{1}{2p_y}\frac{w}{\frac{w - p_x x_0}{2p_y}} = 1 + \frac{p_x x_0}{w - p_x x_0} > 1 \quad \text{(A7)}$$

The income elasticity of the necessity goods is smaller than one, and the income elasticity of luxury goods is greater than one. This result implies that *x belongs to necessity goods, and y belongs to luxury goods*.

Appendix A2

The wealthier agents who consume more luxuries can be described by stating that the consumers are wealthier because they prefer luxuries. We assume that there are two kinds of goods ($i = x, y$) during two periods ($t = 1, 2$). Let c_{it} be the consumption of the goods i in the period t, and let preferences be represented by the additive stationary utility function. In the intertemporal consumption theory, we set that θ is the marginal utility of consumption for the elasticity of substitution, p_x is the normal goods price, p_y is the luxury goods price, and w indicates wealth. We have that

$$u(c_1, c_2) = \frac{c_1^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{c_2^{1-\theta}}{1-\theta} \quad \text{(A8)}$$

subject to $p_x c_1 + p_y c_2 = w$ or $p_y c_1 + p_x c_2 = w$. Applying the method of Lagrange multipliers, we have

$$£ = \frac{c_1^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{c_2^{1-\theta}}{1-\theta} + \lambda(w - p_x c_1 - p_y c_2) \quad \text{(A9)}$$

The first-order condition with respect to c_1 and c_2 implies that

$$\frac{\partial £}{\partial c_1} : c_1^{-\theta} = \lambda p_x \quad \text{(A10)}$$

$$\frac{\partial £}{\partial c_2} : \frac{1}{1+\rho} c_2^{-\theta} = \lambda p_y \quad \text{(A11)}$$

Moreover, we have

$$c_2\left(\frac{1}{1+\rho}\frac{p_y}{p_x}\right)^{\frac{1}{\theta}} = c_1 \Rightarrow \frac{c_1^{-\theta}}{(1+\rho)c_2^{-\theta}} = \frac{p_y}{p_x} \Rightarrow \frac{1}{1+\rho} = \frac{p_y}{p_x}\left(\frac{c_1}{c_2}\right)^{\theta}$$

$$\Rightarrow \left(\frac{1}{1+\rho}\right)^{\frac{1}{\theta}} = \left(\frac{p_x}{p_y}\right)^{\frac{1}{\theta}}\frac{c_1}{c_2} \Rightarrow \left(\frac{1}{1+\rho}\frac{p_y}{p_x}\right)^{\frac{1}{\theta}} = \frac{c_1}{c_2}$$

Taking the logarithm, we have

$$\frac{1}{\theta}\ln(1+\rho) + \frac{1}{\theta}\ln\left(\frac{p_y}{p_x}\right) = \ln\frac{c_1}{c_2} \quad \text{(A12)}$$

The differential $\ln\left(\frac{p_y}{p_x}\right)$ is

$$\frac{\partial\left[\ln\left(\frac{c_1}{c_2}\right)\right]}{\partial\left[\ln\left(\frac{p_y}{p_x}\right)\right]} = \frac{\partial\left(\frac{c_1}{c_2}\right)\left(\frac{p_y}{p_x}\right)}{\partial\left(\frac{p_y}{p_x}\right)\left(\frac{c_1}{c_2}\right)} = \frac{1}{\theta} \quad \text{(A13)}$$

The ratio of relative consumption to relative price shows the relationship $\frac{1}{\theta}$ between normal and luxury goods.

Appendix A3

First, we apply the Shapiro-Stiglitz(1984) model to find an expression for the expected *present value* of revenue the government must raise when $G=G_1$. This is expressed as

$$P_1(\Delta t) = \int_{t=o}^{\Delta t} e^{-rt} e^{-\alpha t}(G_1 + rB)dt + e^{-r\Delta t}\left[e^{-\alpha\Delta t}P_1(\Delta t) + (1-e^{-\alpha\Delta t})P_2(\Delta t)\right] \quad \text{(A14)}$$

The e^{-rt} term indicates the discount with the constant interest rate r, and the first term on the right-hand side of (A14) reflects that the government's revenue must rise during the interval $(0, \Delta t)$. The probability of more expenditures for the government at time t is $e^{-\alpha t}$, in which the government must raise G_1+rB. The second term reflects the revenue necessary after Δt. At that time, the government's purchases are still high with probability $e^{-\alpha\Delta t}$, and it has switched into a low probability $1-e^{-\alpha\Delta t}$. P_1 and P_2 denote the expected *present values* of the revenue the government must raise in each case. And this is discounted by the $e^{-\alpha\Delta t}$ term. The integral in equation (A1) can be solved by the following steps.

First, we know that

$$\int_{t=o}^{\Delta t} e^{-(\alpha+r)t}(G_1+rB)dt = (G_1+rB)\left[\frac{-1}{(\alpha+r)}e^{-(\alpha+r)t}\Big|_{t=0}^{\Delta t}\right].$$

This can be simplified to

$$\int_{t=o}^{\Delta t} e^{-(\alpha+r)t}(G_1+rB)dt = \frac{(G_1+rB)}{(\alpha+r)}\left[1-e^{-(\alpha+r)t}\right] \qquad \text{(A15)}$$

Substitute equation (A15) into (A14), and it yields that

$$P_1(\Delta t) = \frac{(G_1+rB)}{(\alpha+r)}\left[1-e^{-(\alpha+r)t}\right] + e^{-\alpha\Delta t}P_1(\Delta t) + (1-e^{-\alpha\Delta t})P_2(\Delta t).$$

Furthermore,

$$P_1(\Delta t)\left[1-e^{-(\alpha+r)t}\right] = \frac{(G_1+rB)}{(\alpha+r)}\left[1-e^{-(\alpha+r)t}\right] + e^{-r\Delta t}(1-e^{-\alpha\Delta t})P_2(\Delta t),$$

which can be simplified to be

$$P_1(\Delta t) = \frac{(G_1+rB)}{(\alpha+r)} + \frac{e^{-r\Delta t}(1-e^{-\alpha\Delta t})}{1-e^{-(\alpha+r)\Delta t}}P_2(\Delta t) \qquad \text{(A16)}$$

Now take the limit of equation (A16) as Δt goes to zero. The derivative with respect to Δt of the second term on the right-hand side of equation (A16) yields

$$-re^{-r\Delta t} + (\alpha+r)e^{-(\alpha+r)\Delta t} \xrightarrow{\Delta t\to 0} \alpha \cdot$$

The derivative of the denominator of the same term approximates to $\alpha+r$ as $\Delta t\to 0$. That is, $(\alpha+r)e^{-(\alpha+r)\Delta t}\to\alpha+r$.

Thus, as $\Delta t\to 0$, we rearrange and get the expressions of P_1 as follows:

$$P_1 = \frac{(G_1+rB)}{(\alpha+r)} + \frac{\alpha}{\alpha+r}P_2 = \frac{(G_1+rB)+\alpha P_2}{(\alpha+r)} \qquad \text{(A17)}$$

Similarly, it can yield the following expressions for P_2:

$$P_2 = \frac{(G_2+rB)+\beta P_1}{(\beta+r)} \qquad \text{(A18)}$$

We can solve (A17) and (A18) for P_1 and P_2 with the matrix form:

$$\begin{bmatrix} \alpha + r & -\alpha \\ -\beta & \beta + r \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} G_1 + rB \\ G_2 + rB \end{bmatrix}.$$

Applying Cramer's rule to the solution, we have

$$P_1^* = \frac{\begin{vmatrix} G_1 + rB & -\alpha \\ G_2 + rB & \beta + r \end{vmatrix}}{\begin{vmatrix} \alpha + r & -\alpha \\ -\beta & \beta + r \end{vmatrix}} \Rightarrow P_1^* = \frac{(\alpha + r)G_1 + \beta G_2}{r(\alpha + \beta + r)} + B \quad \text{(A19)}$$

and

$$P_2^* = \frac{\begin{vmatrix} \alpha + r & G_1 + rB \\ -\beta & G_2 + rB \end{vmatrix}}{\begin{vmatrix} \alpha + r & -\alpha \\ -\beta & \beta + r \end{vmatrix}} \Rightarrow P_2^* = \frac{\alpha G_1 + (\beta + r)G_2}{r(\alpha + \beta + r)} + B \quad \text{(A20)}$$

Appendix B

Table B1. Data Description and Source

Title	Variable	Data description and source
Luxury taxes with variation	$dT = d\tau$	In the beginning, a luxury tax was imposed in Taiwan from June to December 2011 and aggregated only NT$2.206 billion without tax revenue monthly data. The monthly collection data illustrate that the minimum taxes fell to NT$197.776 million, the maximum taxes rose to NT$513.731 million, and the mean taxes were NT$352.982 million since January 2012.Moreover, the monthly variation dT has been applied as a proxy variable for $d\tau$ since 2012. The data is from the Ministry of Finance, Statistics Database. http://web02.mof.gov.tw/njswww/WebProxy.aspx?sys=100&funid=defjspf2
Consumer price index growth rate (annual)	dp_y	From the decade in Taiwan. In our comprehensive judgment, the construction cost index was 4.95%, and the import price index was 6.1% (in Taiwan, luxury items contain short-term property transactions and import commodities). This article is set at 5%, which is still a reasonable range. The data is from the Directorate General of Budget, Accounting and Statistics, Executive Yuan, Taiwan, Statistics Database. http://www.stat.gov.tw/mp.asp?mp=4