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學術論著

Order from Random Growth Process in the Evolving Complex Systems

動態複雜系統的隨機成長過程中之不隨機

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ABSTRACT

Power law has been shown to be a common feature of many self-organized complex systems, and Zipf's law in regional science is the most famous of all these distributions. This paper shows that the assumption of homogeneity of the random growth process as assumed in Gibrat's law will generate city size distribution as power law. However, Gibrat's law does not necessarily generate Zipf's limiting pattern. City distribution could possibly converge to a Zipf's pattern limiting distribution only with a diminishing decreasing standard deviation of the random growth rate. Moreover, the value of the diminishing rate of the standard deviation of city growth rate determines the speed of the convergence and the value of the converged slope. The homogeneous random evolving process is the essential underlying feature, which generates the common power law property of many complex systems. Nevertheless, the variation of the changing rate of increased potential connections and the sensitivity of interactions among cities are the major reasons for the differences of the slopes among self-organized systems.

Key words: self-organized criticality, complex systems, potential connections,

摘　要

冪次法則(Power law)是自我組織複雜系統的共同特性，而區域科學中的普瑞夫法則(Zipf's law)則是冪次法則的特殊型式。本文以模擬方式顯示，符合吉伯特定理(Gibrat's law)的同質隨機成長假設可導致冪次法則的分配。然而，吉伯特定理不必然衍生出普瑞夫法則的極限分配。在成長率的標準差呈漸緩遞減時，都市成長才可能趨近普瑞夫法則的極限分配。都市成長率標準差的遞減率決定都市大小分配收斂的速度和斜率。都市成長率的標準差與區域內都市間的潛在連繫和都市間交互作用的敏感度有關。

關鍵詞：自我組織臨界值，複雜系統，潛在連繫

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1. Introduction

The world is full of complex systems that are self-organized not only in response to exogenous disturbance, but also in response to internal logic. There is no global controller in the complex system. Many levels of hierarchical organization define the interaction among different level of units. Various deterministic interacted mechanisms in different level of hierarchy and the stochastic factors caused within and outside the system organize the systems' behavior. Global weather, developing embryo, global economy and growing cities are all examples of this self-organizing system. Global economy is composed of many levels of hierarchical organization that consists of various levels of agents and units interacted with each other. The evolution of the global economy results from the evolution of the system. Similarly, the growth of cities within a region, a country or globally is also corresponding to the self-organized mechanism. Cities' growth within a region is caused by both deterministic and stochastic factors. The deterministic factor is based on the systematic features of various hierarchical organizations. Those include the location choices of different agents (residents and firms) and the policy choices of different units; those choices are generated from systematic self-interest optimization decision process. Objects from various self-organizing systems, such as war, sandpiles, earthquakes, forest fires, and city distribution, display a size distribution as power function. Power law has been shown to be a common feature of many complex systems; this common feature reflects certain regularity of the size distribution in different systems. Zip'f law in city distribution is the most famous regularity among all these distributions; it states a linear relation between log of city rank and log of city size of cities' limiting distribution in different countries. This feature indicates a certain limiting hierarchical composition of city distribution and is strongly supported by empirical evidence in different data set. The application of Gibrat's law explains Zipf's pattern by a statistical mechanism. However, this pure statistical explanation is lack of condition for the existence of the limiting distribution and also lack of consideration of the self-organized property and economic foundation.

The purpose of this paper is to investigate the possible underlying mechanisms or properties in complex systems that would lead to this general simple regularity given different structures. Furthermore, to explain power law and Zipf's law from the self-organized complex system point of view that has not been taking into account to explain Zipf's pattern before.

Section 2 introduces the essential idea and property of the dynamical complex systems. Section 3 gives a brief review of power law, rank-size rule and Zipf's law which describe urban size distribution in regional studies. Section 4 shows the common features of the self-organized criticality: power law. Section 5 investigates the emergence of power law and Zipf's law theoretically and empirically both from the stochastic process and self-organized feature point of views. The conclusion is in Section 6.

2. The Evolving Complex System

Philip Anderson, the Nobel laureate physicist defined the complex system as a science of "emergence". It is about the surprising ensembles from the nonlinear combination of the interacting units in the system. From the structure point of view, the complex system is self-organized as systems form from almost random or homogeneous state based on some deterministic rules. Complexity is also defined as a measure of the sensitivity of particles in the systems. It is the potential connections among agents in the systems. The complex system is first observed in physics and biology. Further research about economy as an evolving complex system is discussed in Santa Fe Institute in 1987. The nonlinear dynamic structure among the interacting units results in several special features.

Authur, Durlauf and Lane (1997) pointed out six features of the complex system: no global controller, dispersed interaction, crosscutting hierarchical organization, continual adaptation behaviors, perpetual novelty niches, and out-of-equilibrium dynamics. The organized units in the systems are interacted with each other in a very pluralistic form. There are many levels of hierarchical organization defining the interaction among different level of units. Various deterministic interaction mechanisms in different level of hierarchy organize the systems' behavior. There is no global controller in the system. In addition, the various deterministic interaction rules are adaptive, and the system is perpetual novelty. The most important feature that is very much different form the classic economic theory is the out of equilibrium features. There is no unique optimum or global equilibrium in the system.

Systems with these features are called adaptive nonlinear networks. Due to the structural features of the system, there are some essential properties of this adaptive nonlinear network. The possible evolving outcomes of the system is path dependent; the historical shock is very crucial for the result. Also, it shows "lock-in" property in the evolution process. Once an "alternative" is chosen, it is difficult to exit (named as lock-in effect) due to the increasing return to scale property. Thus, the evolving result is possible to be inefficient as the whole or at the end. Even there is a better alternative later, the system might stock in the less efficient alternative due to the increasing return to scale property. The outcome of the system is not predictable and multiple equilibria are possible.

The evolving outcome is very sensible to the initial conditions and the value of parameters. Different value of the parameters will result in different characteristic of the evolving process. Some evolving process is predictable and stable, but some are unpredictable and unstable. Features of this self-organized complex system have been applied to study various economic phenomena, such as positive feedback in economics, the historical path dependence in urban systems, nonlinear theory in global business cycle, input and output structure in the percolation economics, and the financial feedback in market mechanisms (see Arthur 2000; Krugman 1996; Day 1994). Urban systems are composed of different levels of hierarchy units whose location and policy decisions form the systematic part of the system behaviors.

Krugman (1996) suggested two principles of self-organized process in explaining economic system: order from instability and order from random growth. The first principle indicates the astonishing empirical evidence that simple order emerges from the unstable self-organized criticality that is generated from the out-of equilibrium dynamical systems. The second principle states the possible explanation for the feature of the emergence of order. Gibrat's law is a typical example. This principle will be further investigated.

3. Zipf's law and Gibrat's law

Pareto distribution is most commonly employed to study urban sizes in regional research (see Mills and Hamilton 1994):

$$G(x)=Ax^{-a} \quad \text{.......... (1)}$$

where $G(x)$ is the number of cities with at least x people; it could be interpreted as the rank of the city with x people. Variable is the city size; parameters A and x are constants to be estimated from the data (cited from Mills and Hamilton 1994, p.78). Relation in equation (1) is also called power law, which describes the number of cities with a population larger than x is proportional to x^{-a} (see Fujita, Krugman and Venables 1999, ch. 12). The power law with the exponent, x, close to one, is referred to as Zipf's Law. The alternative name is the rank-size rule (see Fujita, Krugman and Venables 1999, p.217):

$$G(x)=Ax^{-1} \quad \text{.......... (2)}$$

It refers the following relation:

$$xG(x)=A \quad \text{.......... (3)}$$

Zipf's law (or rank-size rule) proposes that the product of the city size (x) and its rank (G(x)) is a constant. Take the log of the city size and city rank in equation (2):

$$In(Rank)=A'-In(Size) \quad \text{.......... (4)}$$

where $A'=In(A)$; it is a constant. Equation (4) is an alternative expression and most commonly employed to present Zipf's law (or rank-size rule). This relation has robust empirical evidence across countries and time. When an urban system has city distribution as Zipf's law, the constant A in equation (3) represents the population of the largest city in the region. Consequently, the second-largest city would have one-half the population of the largest, and the third, one-third that population, etc. This distribution regularity in various urban systems are based on different levels of economic decision processes.

Gabaix (1999) proposes Gibrat's law to describe Zipf's pattern in city distribution. He states that if cities grow randomly with the same expected growth rate and the same standard deviation, the limiting distribution will converge to Zipf's law (Gabaix 1999). Gibrat's law indicates that when cities' size randomly grows with a homogeneous growth process[1], the limiting distribution will converge to Zipf's pattern regardless of the distribution type of the growth rate.

4. The common order from the self-organized criticality: power law

The hierarchical structures and the interaction among agents generate the complex evolution

[1] The expected growth rate and standard deviation of city growth rate are the same across cities; they are independent of city size.

process. Per Bak et al. demonstrated that dynamical systems naturally evolve into highly interactive critical states which are barely stable (see Bak, Tang, and Wiesenfeld 1987). This self-organized criticality, where a small cause may lead to a large event, is the common underlying mechanism behind the special feature: the power law distribution of the corresponding events (see Bak and Paczuski 1995). The events, named as the complexity cascades of self-organized criticality in social system, take the form of wars, strikes, economic depressions, collapses of government, coalitions, emergence of cities, and many others. The historical details of these events are unpredictable; however, the statistical distribution of these events is predictable. This predictable power law appears as a straight line in a double logarithmic plot of rank and size. It is empirically observed in various complex systems: ecological systems, social systems and geophysical phenomena. They are organized by various mechanics and networks type, but all results in characteristic power function in the size distribution.

Brunk (2000) propose that the evolution process of the complex system characterized by the self-organized criticality is composed of two parts: a systematic growth factor and a random growth factor. The systematic growth factor is characterized by the underlying mechanism and depends on the current degree of "complexity" of the systems. The complexity is measured as the degree of the potential connection among members, and is growing as the systems getting larger or extending more subsystems. The increasing complexity implies increasing in the sensitivity of interactions. A random growth factor could be generated either inside or outside the systems. These systematic and unsystematic growth factors together with the increasing complexity property in the complex systems are essential in explaining power function phenomenon. The most robust evidence of power function in complexity cascades appears in urban system.

Zipf (1949) found that city sizes follow a very simple distribution law. He showed that the size distribution of cities satisfies power law with a scaling exponent equals one empirically for many different societies and time periods. It is named as Zipf's law showing that the probability the size of a city greater than some size x is proportional to 1/x. The expression of Zipf's law can be visualized by taking a country cross sectional data on city size and city rank. The plot with the log of rank along the y-axis, and the log of the population along the x-axis will mostly show a straight line with slope very close to -1.

The empirical evidence of Zipf's law is shown in various data sets: most modern countries (see Rosen and Resnick 1980), India in 1911 (see Zipf 1949), U.S. history (see Dobkins and Ioannides 1998; Krugman 1996; Zipf 1949) and China in the mid-nineteenth century (see Rozman 1990). More recent data shows that Zipf's law remains rather good approximation for developed countries. However, cities in countries with a unique social structure, such as the former USSR or China, do not quite follow Zipf's law (see Marsili, and Zhang 1998).

Empirical evidences show that individual decision rules followed by the underlying hierarchical structure do influence the generating process of self-organized criticality. The growth of the cities in the region is characterized by the hierarchical system and is self-organized. Gabaix (1999) proposes Gibrat's law to describe Zipf's law in city distribution. However, Gabaix's work only shows that if there is a steady state process of the tail distribution of city sizes, the limiting distribution would be as Zipf's pattern. He did not discuss the condition for the existence of the steady state process.

This finding of Gibrat's law suggests a pure statistical view in explaining Zipf's law other than the usual economics views based on the urban theories. According to the urban theories, agents in the space system (residents and firms) make their location decisions depending on their corresponding economics considerations to optimize their own goals. The major consideration in location decisions includes the location accessibility and the agglomeration effects. The agglomeration effect could be measured as the sum of agents (residents or firms) in each city weighted by its corresponding location accessibility. The larger the agglomeration effects the larger the location attractiveness; consequently, the faster this place would grow. According to this explanation, it seems counter intuitive that city growth is independent of city size as Gibrat's law proposing. In the next section, we will investigate the possible explanation for the Zipf's law concerning self-organization features, location theory and Gibrat's law.

5. A model and the simulation about power law

5.1 The model

According to Gibrat's law (Gabaix 1999), Cities' limiting distribution will converge to Zipf's pattern if they grow randomly with the same expected growth rate and the same standard deviation.

One way to present the distribution of randomly growing cities is by Joseph Steindl (1965): Consider a region consists of n zones. Let $x_{i,t}$ denotes the normalized size of zone i at time t, which is the population of city i divided by the total region population. The growth of city size is proportional to the current city size. The growth rate of zone i at time t, $\varepsilon_{i,t}$, is identically independent distributed with mean zero and variance $\sigma_{i,t}^{\ 2}$. The growth process of the normalized size of city i at time t is as follows:

$$x_{i,t}-x_{i,t-1}=\varepsilon_{i,t}x_{i,t-1} \quad (5)$$

$$x_{i,t}=(1+\varepsilon_{i,t})x_{i,t-1}=x_{i,0}(1+\varepsilon_{i,1})(1+\varepsilon_{i,2})...(1+\varepsilon_{i,t}) \quad (6)$$

The sum of the normalized size of all cities in the region at certain time period is one: $\sum_i x_{i,t}=1$. Consequently, the average normalized size at each time period stays constant through time. Assuming the time period is so short that the growth rates, $\varepsilon_{i,t}$, is relatively small. This justify the following approximation:

$$log(1+\varepsilon_{i,t})\cong\varepsilon_{i,t} \quad (7)$$

Taking logs of equation (6); it becomes:

$$logx_{i,t}=logx_{i,0}+\varepsilon_{i,1}+\varepsilon_{i,2}+...+\varepsilon_{i,t} \quad (8)$$

The term $logx_{i,0}$ would be very small comparing to the term $logx_{i,t}$ as $t\rightarrow\infty$, the distribution of is approximately normal distribution with mean zero and variance $s_t^{\ 2}$ when the growth rate of cities at time t have the same variance s_t^2 across cities. Thus, the distribution of normalized city size $X_{i,t}$ is lognormal.

In addition, city rank can be expressed as the tail distribution of city sizes at time t:

$$R_t(X)=P(x_t>X) \quad (9)$$

As $t \rightarrow \infty$, the observations of $logR(X)$ versus $logX$ would lie in a straight line with negative slope if the distribution of normalized city size were lognormal. This represents the power pattern limiting distribution:

$$R(X)=\alpha/X^{\beta} \quad (10)$$

This is the power law feature that is common in many evolving complex systems. Generally, a group of entities with homogeneous random growth rate will generate power law limiting distribution if there is steady state process.

An alternative method in Gabaix (1999) is applied here to investigate the possible limiting distribution given a homogeneous random growth process. Assume the city growth process is as in equation (5). It follows:

$$x_{i,t}=(1+\varepsilon_{i,t})x_{i,t-1}=\gamma_{i,t}x_{i,t-1} \quad (11)$$

where $\gamma_t=1+\varepsilon_t$, and $x_{i,t}$ denotes the normalized size of zone i at time t. The tail distribution of city sizes at time t (equation (9)) can be expressed as (Gabaix, 1999):

$$R_t(X)=P(x_t>X)=\int_0^{\infty} R_{t-1}(\frac{X}{\gamma_t})f(\gamma_t)d\gamma \quad (12)$$

Assume a general distribution of the type $R_t(X)=\alpha/X^{\beta(t)}$, we could derive a general term from equation (12):

$$R_t(X)=\alpha X^{-\beta(0)}E[\gamma_1^{\beta(0)}]E[\gamma_2^{\beta(1)}]...E[\gamma_t^{\beta(t-1)}] \quad (13)$$

The condition of the existence of a steady state process $R_t(X)=R(X)$ is: $\lim_{t\rightarrow\infty} E\left[r_t^{\beta(t-1)}\right]=1$. A constant, $\beta(t)=\beta$, will do. The value of parameter β is not necessarily to be one to assure the existence of a steady state limiting distribution. This indicates that if there is steady state process, the limiting size distribution is as power function, not necessarily as Zipf's law. This finding is different from statement of Gabaix (1999) that the limiting distribution would be Zipf's law.

5.2 Simulation

The simulation experiments in this section is based on the homogeneous growth process assumed in Gibrat's law. Cities randomly grow with the growth rate of the same expected mean and standard deviation. The growing process is independent of city size. A homogeneous growth process across cities includes two types: (1) homogeneous across cities and time, and (2) homogeneous across cities only. Both types of growth process will be simulated in this section.

In experiment 1, we assume the first type of growth process that cities grow with the same expected mean and standard deviation across cities and time. The standard deviation of the random growth factor is assumed constant across cities and time: $\sigma_{i,t}{}^2 = \sigma^2$, where $\sigma_{i,t}{}^2$ represents the variance of the growth rates of city i at time t. The variance of the random growth rate indicates the degree of growth stability of certain city at certain time. If cities in the region grow with different variance ($\sigma_{i,t}{}^2$), it shows that cities grow with different degree of stability. Cities with smaller variance of growth rate will have growth pattern more stable than cities with larger variance of growth rate. If cities in the urban system grow with the same variance ($\sigma_t{}^2$), the variance of the random growth rate represents the degree of the heterogeneity among cities' growth. The larger the variance, the more diverse cities grow; on the contrary, the smaller the variance, the more evenly cities grow.

The evolutions of cities in the urban system are interacted with both competitive and cooperative relationships. The growth of city in the urban system reflects the location advantage from the interaction among cities in the system. Thus, the heterogeneity among cities' growth mainly refers to the relative location advantages or disadvantages among cities. A constant variance of the random growth rate across cities and time (σ^2) indicates that the differences among cities' location attractiveness remain the same across time. However, in the real world, the differences among cities' location attractiveness tend to change across time due to the change of various economic phenomenon. We simulate this urban system and estimate the slope of equation (14) to study its evolving process.

$$In(Rank)=A'+a\ ln(Size) \quad \text{.......} \quad (14)$$

This relation is derived from pareto distribution in equation (1) of cities distribution. The sign of the slope (a) is negative due to power law; the absolute value of the slope indicates the degree of diverse in cities' size. A larger absolute value of the slope implies that size of cities is more evenly distributed; a smaller absolute value of the slope implies a more heterogeneous size distribution.

In this experiment, we simulate city growth given homogeneous random growth rate across cities and time. The parameter values and the simulation processes are presented in Appendix. The estimated slopes (a) in equation (14) given different value of standard deviation of the random growth rate are presented in Table 1. All the estimated slopes are negative as pareto distribution describes. All estimated absolute value of slopes given different value of standard deviation decrease as time pass. This denotes that cities in the urban system evolve into more and more heterogeneously distributed. In other words, sizes of cities are getting more and more diverse as time pass by. Furthermore, the estimated absolute value of the slope is getting smaller as time pass without a limit boundary. This result indicates that cities would continue heterogeneously grow without a limiting distribution given the homogeneous random growth rate across locations and time. A constant slope indicates a power law distribution. As the slope equals -1, the distribution fulfills Zipf's law.

This finding is inconsistent with Gibrat's law. Given the assumption of Gibrat's law, Gabaix only shows that if the steady state exists, Zipf's pattern would be the limiting distribution in the steady state. In other words, he shows that Zipf's limiting distribution is the possible result if there is steady state given the homogeneous growth process. Gibrat's law does not assure that Zipf's distribution is the necessary condition of the homogeneous growth process. The simulation shows that the assumption in Gibrat's law does not assure the existence of the steady state in size distribution; however Zipf's pattern emerges only when a steady state process exists. We would like to discuss further the possible condition for the existence of a steady state process given the assumption in Gibrat's law.

As we mentioned earlier, cities' random growth rate with a smaller standard deviation (σ^2) denotes less difference among cities' location attractiveness in the region; thus, cities' growth is less various; cities' size is more evenly distributed. The less the differences of the location attractiveness among cities, the larger the absolute value of the regression slope (a). On the contrary, a random growth rate with a larger standard deviation indicates larger differences among cities' location attractiveness in the region; cities' growth is more diverse; cities' size is more heterogeneously distributed; and the estimated slope is flatter (Table 1).

Table 1 Estimated slope (a) in Simulation of Cities Growth with Constant Variance (σ^2) $\ln(Rank) = A' + a \ln(Size)$

Experiment	(1)	(2)	(3)
σ	0.01	0.05	0.1
S1	-10.81	-2.39	-1.36
S2	-7.96	-1.70	-0.82
S3	-7.17	-1.33	-0.67
S4	-6.12	-1.12	-0.59
S5	-5.10	-1.16	-0.53
S6	-4.59	-1.02	-0.50
S7	-4.31	-0.99	-0.44
S8	-3.95	-0.87	-0.42
S9	-3.74	-0.84	-0.41
S10	-3.57	-0.79	-0.39

Notes: The number of cities: 100.
The experimental time periods: 500.
σ: The standard deviation of the random growth rate.
S1: The estimated slope of the regression of log rank against log size at time t=50.
S2: The estimated slope of the regression of log rank against log size at time t=100.
S3: The estimated slope of the regression of log rank against log size at time t=150.
S4: The estimated slope of the regression of log rank against log size at time t=200.
S5: The estimated slope of the regression of log rank against log size at time t=250.
S6: The estimated slope of the regression of log rank against log size at time t=300.
S7: The estimated slope of the regression of log rank against log size at time t=350.
S8: The estimated slope of the regression of log rank against log size at time t=400.
S9: The estimated slope of the regression of log rank against log size at time t=450.
S10: The estimated slope of the regression of log rank against log size at time t=500.

In this evolving urban system, agents choose cities to locate according to their decision rule one by one and period by period. There is no global controller designing the growth of the region. It is all determined by the dispersed interaction among agents. Also, according to the adaptation behaviors within the hierarchical structures in the system, the potential connections and interactions among agents and locations are getting more and more frequent and sensitive during the evolution process. More frequent interactions and increasing sensitivities among locations and agents would lead to less variance of the relative location attractiveness among cities. Thus, this growing sensitivity feature could be characterized by a decreasing standard deviation of cities' random growth rate. However, the increase of the sensitivities and connections among agents and locations are reducing and converging to a critical state. This feature could be characterized as a diminishing decrease of the standard deviation of cities' random growth rate.

Cities grow based on the growth rate with a diminishing decreasing standard deviation is simulated in experiment 2. The simulation results are presented in Table 2. Assume that the standard deviation of the random growth factor, σ_t, is decreasing in a reducing rate across time. We use a simple form to express this standard deviation:

$$\sigma_t = \sigma_0 t^{-b} \qquad (15)$$

Where the parameter, b, represents the reducing rate of the standard deviation. This equation is a simple model reflecting a diminishing decreasing standard deviation. A growth rate with a constant standard deviation across cities and time will generate an urban system with cities evolving into more and more heterogeneously distributed without limiting distribution; the absolute value of the slope in equation (14) is decreasing. A growth rate with a decreasing standard deviation without boundary across time will generate an urban system with cities evolving into less and less heterogeneously distributed in a constant rate; the absolute value of the slope in equation (14) is decreasing with a limiting value. A growth rate with a diminishingly decreasing standard deviation across time will generate an urban system with cities evolving into less and less heterogeneously distributed in a diminishing rate; the absolute value of the slope in equation (14) is decreasing to a limiting value in a diminishing rate.

Given the same initial condition, a larger diminishing rate (b) denotes the standard deviation, σ_t, decreases faster across time. The faster the standard deviation decreases, the faster the dispersion of the growth rate among cities is reduced; consequently the slower the speed of changing into heterogeneous city size distribution is; finally, the faster the urban system converge to a limiting distribution. By the way, the larger the diminishing rate of the standard deviation, the smaller the value of the standard deviation, and the larger the absolute value of the slope. On the contrary, a smaller diminishing rate indicates that cities' size distribution is more heterogeneous distributed and converge slower. These features are observed in Table 2.

The random growth rate with a diminishing decreasing standard deviation across time affects the reduction speed of the regression slope. As the potential connection and sensitivity among cities increase across time, the diverse of the relative location advantage among cities decrease, and consequently the variance of the growth rate across cities decrease. The speed of cities' evolution from uniform to

Table 2 Estimated slope (a) in Simulation with Decreasing Variance (σ_t^2)*
ln(*Rank*)=A'+*a* ln(*Size*)

Experiment	(1)	(2)	(3)
σ_0	0.1	0.1	0.1
b	1	0.23	0.1
S1	-5.918	-1.881	-1.543
S2	-5.903	-1.566	-1.181
S3	-5.898	-1.420	-1.116
S4	-5.877	-1.283	-0.989
S5	-5.860	-1.203	-0.884
S6	-5.856	-1.208	-0.805
S7	-5.841	-1.164	-0.723
S8	-5.839	-1.121	-0.664
S9	-5.843	-1.097	-0.611
S10	-5.836	-1.097	-0.593

* $\sigma_t = \sigma_0 t^{-b}$

Notes: The number of cities:100.

The experimental time periods: 500.

S.D.: The standard deviation of the random growth rate.

S1: The estimated slope of the regression of log rank against log size at time t=50.

S2: The estimated slope of the regression of log rank against log size at time t=100.

S3: The estimated slope of the regression of log rank against log size at time t=150.

S4: The estimated slope of the regression of log rank against log size at time t=200.

S5: The estimated slope of the regression of log rank against log size at time t=250.

S6: The estimated slope of the regression of log rank against log size at time t=300.

S7: The estimated slope of the regression of log rank against log size at time t=350.

S8: The estimated slope of the regression of log rank against log size at time t=400.

S9: The estimated slope of the regression of log rank against log size at time t=450.

S10: The estimated slope of the regression of log rank against log size at time t=500.

heterogeneous is reducing. As the potential connections and sensitivity of interactions among cities are increased in a diminishing rate, the speed of cities' evolution into heterogeneous distribution is reduced and hopefully the urban system converge to a limiting pattern given certain parameter value. Furthermore, the converged slope is closely related to the speed of the decreasing rate of the variance of the growth rate.

Simulation result in Table 2 shows that a diminishing decreasing standard deviation of the growth rate under Gibrat's law possibly lead to a convergence of size distribution given certain parameter value. That is, this further condition of the standard deviation of growth rate given Gibrat's law may result in a steady state process. However, it does not promise the limiting pattern in the steady state process as Zipf's pattern. Zipt's pattern only emerges given certain values of parameters. We could conclude that given the assumption in Gibrat's law, a diminishing decreasing standard deviation of the growth rate is the necessary condition for the existence of a steady state process and a limiting distribution.

6. Conclusion

Power law has been shown to be a common feature of many complex systems, and Zip'f law in regional science is the most famous of all these distributions. The application of Gibrat's law explains Zipf's pattern by a statistical mechanism. However, this pure statistical explanation is lack of condition for the existence of the limiting distribution and considerations from self-organized process. This paper is the first work investigating the source of Zipf's law from the complex system point of view. It shows that if region grows based on random growth rates with the same mean and variance across cities, it will generate power law distribution of city size. However, a random growth process as assumed in Gibrat's law does not necessarily generate Zipf's limiting pattern. According to the features of the dynamic complex systems, the adaptation behaviors of the interacted agents under the hierarchical structures in the system will lead to more frequent potential connections among agents and higher sensitivity of interactions. As the potential connections and sensitivity of interactions among agents and cities increase into a critical state across time, it reflects the difference of the location attractiveness among cities in the region is reducing in a diminishing rate. This could be characterized by a growth rate with diminishing decreasing standard deviation. The simulating result shows that a diminishing decreasing standard deviation of the random growth rate could possibly generate a Zipf's limiting distribution given certain parameter value. Moreover, the value of the diminishing rate determines the speed of the convergence and the value of the converging slope.

Generally, this finding also explains the power law features of the other complexity cascades from the corresponding self-organized complex systems. System particles with homogeneous random evolving rate would generate the power function distribution. A homogeneous random evolving process is the essential underlying feature, which generates the common power law property of many complex systems. Nevertheless, the major reason that the slopes of power functions differ among various self-organized critical systems is the variation of the changing rate of the increased potential connections and sensitivity of interaction within the systems. In other words, the converging slope of the power function is highly related to the changing rate of the increment of potential connection and the level of complexity.

Our finding suggests that homogeneous growth process does not assure the existence of the Zipf's pattern distribution. Cities grow with the random growth process with diminishing decreasing standard deviation would lead to a convergence of city distribution. The diminishing rate and the initial value of the standard deviation would affect the pattern of the limiting distribution. That is, Zipf's limiting distribution will appear given a diminishing decreasing standard deviation growth rate with certain value of parameters.

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Appendix

1. Experiment 1 (Table 1): Cities' growth rate with constant standard deviation

Simulation parameter:

Number of cities in the region: 100

The experiment time periods: 500

The observed time periods: 50

The initial city size: 1

Cities in the region randomly grow with normal distribution

(Mean=0, Variance = σ^2, σ=0.01, 0.05, and 0.1)

Simulation process:

(1) Generate growth rate by randomly drawing from a normal distribution with mean zero and each of the assumed constant standard deviation (0.01, 0.05, and 0.1)

(2) In period 1, 100 Cities grow according to the randomly drawing growth rate in step 1 given the same initial population (initial city size =1). Derive 100 cities size at period 2.

(3) In period 2, 100 Cities continue growing from period 1 according to the randomly drawing growth rate in step 1. Derive 100 cities size at period 2.

(4) Repeat step 3 until period 50; estimate the slope by regressing ln(rank) versus ln(size).

(5) Repeat step 4 until period 500.

2. Experiment 2 (Table 2): Cities' growth rate with diminishing decreasing standard deviation

Simulation parameter:

Number of cities in the region: 100

The experiment time periods: 500

The observed time periods: 50

The initial city size: 1

Cities in the region randomly grow with normal distribution

(Mean=0, Variance = σ_t^2, $\sigma_t = \sigma_0 t^{-a}$)

(σ_0, a)=(0.1, 1), (0.1, 0.23) and (0.1, 0.1)

Simulation process:

(1) Generate growth rate by randomly drawing from a normal distribution with mean zero and each value of the assumed decreasing standard deviation ($\sigma_t = \sigma_0 t^{-a}$)

(2) In period 1, 100 Cities grow according to the randomly drawing growth rate in step 1 given the same initial population (initial city size =1). Derive 100 cities size at period 2.

(3) In period 2, 100 Cities continue growing from period 1 according to the randomly drawing growth rate in step 1. Derive 100 cities size at period 2.

(4) Repeat step 3 until period 50; estimate the slope by regressing ln(rank) versus ln(size).

(5) Repeat step 4 until period 500.