

An Extension to Shyy's "Real Estate Pricing in a Forward Market"

遠期市場下不動產定價模型的延伸

黃達業*

Dar-Yeh Hwang*

摘 要

本文應用複式選擇權(Copoung Option)概念延伸史綱(1992)的房地產定價模型。首先，以簡單套利推論(Arbitrage Argument)聯結預售屋與成屋價格之關係來檢視史綱模型。然後，應用複式選擇權來評估買者的買權(buyeer's call option)因而導出比史綱更一般化的預售屋定價模型。
(關鍵詞：預售屋、複式選擇權、套利推論、買者的買權)

ABSTRACT

By applying the compound option, this note extends Shyy's (1992) model to price real estate in a forward market. First, I critically review Shyy's model using simple arbitrage arguments to relate the pre-sales and existing house prices. Then, I apply the compound option model to evaluate the buyer's call option and hence derive the generalized version of Shyy's pricing model for the pre-sales housing.

(Keywords: Pre-sales House, Compound Option, Arbitrage Argument, Buyer's Call Option)

(本文於1996年3月27日收稿，1996年5月10日審查通過)

* 台灣大學財務金融系教授

* Professor, Department of Finance, National Taiwan University, Taiwan, R.O.C.

1. INTRODUCTION

Pre-sale housing investment is a popular real estate selling system for new housing in Taiwan, Korea and more recently in Mainland China. The pre-sale system is similar to some derivative contracts, including forwards, futures and options. Since the delivery time of the underlying asset in the pre-sale system is the contract's maturity date, one can construct the pre-sale property pricing model by applying the model for pricing forward/futures contract. Since the process of arbitrage should ensure that the forward/futures price equals the current spot price plus the carrying charge, the effective pre-sale price should also equal the current house price plus the carrying charge (Shyy [1992], and Chang and Ward [1993]). This note extends Shyy's (1992) model by applying both the forward pricing model and the compound option pricing model to examine the case of installment payments and the buyer's option to default.

Let's first define the following notations. $S(t)$ is the spot price of the existing house at time t ; $F(t)$ is the price of the pre-sales housing at time t ; c is net rent received per period; r is the annual interest rate for the time deposit; T is the date for delivering pre-sale contract; t is the current date of pre-sale; d is the depreciation rate on existing house including maintenance costs; F is the proportion of total amount for the pre-sales housing price paid at the initial time; a is the proportion of total amount for the pre-sales housing price paid as the installment over the contract period. Also, let $CC(r,t,T,\delta,\Phi)$ represent the carrying cost and $CR(c,t,T)$ represent the carry return. Then, the theory of carry cost says that $F(t) = S(t) + CC(r,t,T,\delta,\Phi) - CR(c,t,T)$, and one can apply the arbitrage argument to evaluate the pre-sales housing (Shyy[1992], and Chang and Ward [1993], and Chang [1994]).

Suppose we have two alternative investment strategies, A and B. For the investment A, the investors can buy the existing house, and rent it for the rental revenue and hold until the maturity date (T) for delivering pre-sales housing, while for the investment B, the investors can invest in pre-sale housing and put all the cash in deposit account and earn the interest income until the delivering date (T). Here, I assume that the characteristics of pre-sale and existing house are the same. If the value of these two portfolio is not equal, then arbitrage opportunity will exist. The investors will eliminate these arbitrage profits by short selling the higher value one and buying the low value one. Since the depreciation expense, the installment payment and other factors including buyer's call option will affect the price of the pre-sales price, I will examine each factor consequently (Shyy [1992], and Chang [1994]).

With no depreciation, the payoff for the investment A at the maturity date will include the price of the existing house $S(T)$ plus the total rental income in the manner of continuously compounding $(S(t)(e^{rt}-1))$ where $\tau=(T-t)$ subtracted from the investment cost $S(t)$. The payoff for the investment B will be the price of pre-sale housing at delivering date T plus the interest income from time deposit subtracted from the initial price of the pre-sales housing. Under no arbitrage condition, one will have the same payoff from these two alternative investment strategies. Since I assume no depreciation for the existing house, then at the maturity date, both prices of existing house and pre-sale housing should be the same, i.e., $S(T)=F(T)$. Hence, one will have the following relationship

$$S(T) + S(t) (e^{rt}-1) - S(t) = S(T) + S(t) (e^{rt}-1) - F(t) \dots\dots\dots (1)$$

From equation (1), one can obtain the pricing formula for the pre-sales housing as

$$F(t) = S(t) + S(t) (e^{rt}-e^{ct}) \dots\dots\dots (2)$$

From equation (2), one can see that the price of pre-sales housing depend on the current price of the existing house, the annual interest rate on time deposits, the annual rental rate and the maturity date/delivering date (T). If the annual interest rate is higher than the rental rate, then the price of pre-sales housing should be higher than that

of existing house. Otherwise, if the interest rate is lower than the rental rate, then the price of pre-sales housing will be lower than that of existing house.

Next, the price of existing house with depreciation rate d at the maturity date $S(T)$ will be $F(T)-S(t)e^{\delta T}$. Hence, the price of pre-sales housing will be

$$F(t) = S(t) + S(t) (e^{\delta T} + e^{rT} - e^{cT} - 1) \dots\dots\dots (3)$$

From equation (3), one can demonstrate that the basis could be positive or negative depending on the size of interest rate, depreciation rate as well as the rental rate. If the last term in equation (3) is negative, then the price of pre-sales housing will be lower than that of existing house. Otherwise, it will be higher than that of existing house.

Further, with depreciation rate 08/09/96 and initial payment Φ , the investor must pay part of the price as the initial installment amount in order to buy the pre-sales housing. Then, the price of the pre-sales housing will be

$$F(t) = S(t) + S(t) [e^{\delta T} - e^{cT} + (1 - \Phi)(e^{rT} - 1)] \dots\dots\dots (4)$$

From equation (4), we can see that the price of the pre-sales housing will be affected by the initial installment payment (Φ), the interest rate on time deposit (r), the depreciation rate (δ), the annual rental rate (c), the maturity date for delivering the pre-sales housing (T), and the current price of existing house. Again, whether the price of pre-sales housing will be higher or lower than that of existing house depends on the annual interest rate for the time deposit, net rent received per period, and the depreciation rate on existing house including the maintenance costs.

Finally, with depreciation rate δ , initial payment Φ , and two installment payments, then after the investor pays the first initial installment payment, he needs to pay two more installment payments, one in the middle of the construction stage (i.e., at time $\tau/2$), the other at the completion stage of the construction project (i.e., at time T). Then, the price of the pre-sales housing will be

$$F(t) = S(t) + S(t) [e^{\delta T} - e^{cT} + (2 - 2\Phi - \alpha/2)(e^{rT/2} - 1)] \dots\dots\dots (5)$$

Also, one can generalize the above case by paying each month $\alpha/N S(t)$ for N months ($N=12(T-t)$) such that the payoff for his investment strategy B will be $F(T) + (1 - \Phi)S(t)(e^{r/12} - 1) + N(1 - \Phi - \alpha/N)S(t)(e^{r/12} - 1) - F(t)$. Hence, the price of the pre-sales housing will be

$$F(t) = S(t) + S(t) \{e^{\delta T} - e^{cT} + [N(1 - \Phi) - (N - 1)\alpha/2](e^{r/12} - 1)\} \dots\dots\dots (6)$$

Equation (6) represents the general model for pricing the pre-sales housing considering depreciation, initial payment and N installment payments (still without considering the buyer's call option, developer's reputation and transaction tax, etc.) (Shyy [1992], Chang and Farr [1993], and Chang and Ward [1994]).

2. VALUATION OF BUYER'S CALL OPTION

Since after the start of the construction project and before the delivery of the pre-sales housing, the investor of pre-sales housing has the right to default when the price of the existing house falls sharply (i.e., when the price of the existing house falls below the exercising price of the pre-sale house) without going into any lawsuit problems. Then, one can coin this right as call option on the existing house (Shyy [1992]). Since one has two installment payments, one will have two options. At time $(T-t)$, one can keep the pre-sale contract alive by paying the first installment payment under the condition that the investor expect he will be in-the-money. At the delivery date, the investor will pay the final payment if he finds that he is in favorable position by the exercising the option (see figure 1). Hence, the value of this call option on call option can be priced by the compound call option valuation

model (Geske [1979]). Under this case, one will have N installment payments and hence have $(N-1)$ compound options (see figure 2).

In order to apply the compound option pricing model, I make the following simplified assumptions as Black-Scholes (1973) did. (1). The short-term interest rate is known to be the annual interest rate for the time deposit. (2). The price of existing house (S_t^2) follows a random walk in continuous time with a variance rate (σ^2) proportional to

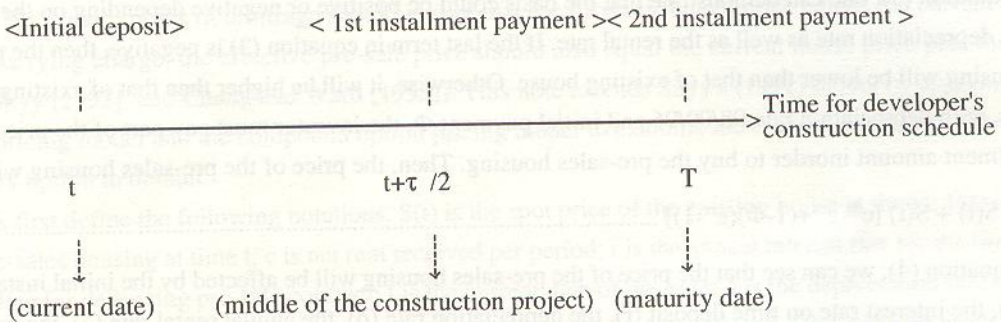


Fig. 1 The buyer's compiuond option in 2 installment payment cases

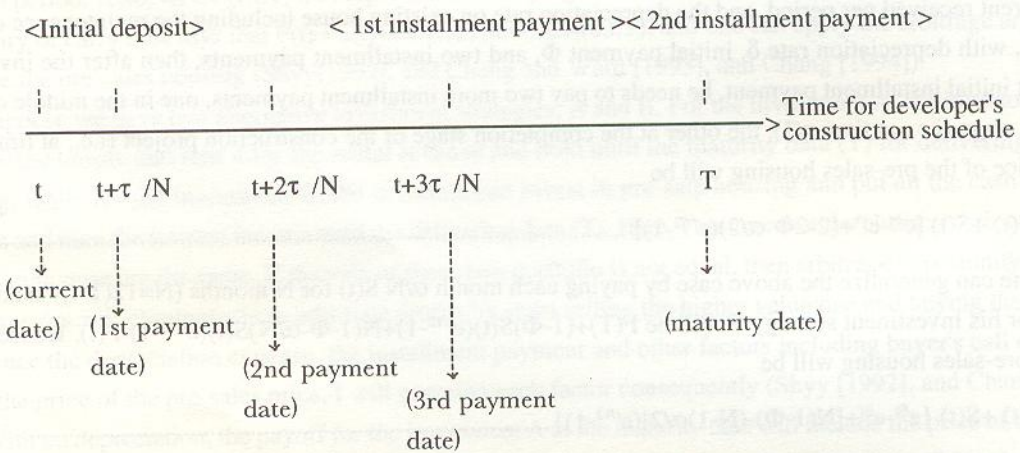


Fig. 2 The buyer's compiuond option in N installment payment cases

the square of the existing house price. Thus, the distribution of possible existing house prices at the end of any finite interval is lognormal with constant variance rate of the return on the existing house. (3) The existing house will deteriorate at the depreciation rate δ . (4) The pre-sale housing is delivered at price of X only in existing date. (5) Unlimited borrowing at short term interest rate is possible. (6) There are no transaction costs in buying or selling the existing house or the pre-sale housing. (7) There are no penalties to short selling. (8) Both the investors and developers are risk-neutral. The critical assumptions among these are (2) and (8). The buyer's call option in this analysis is assumed to be European-style with exercise prices C_t^* where C_t^* in 2 installment payments case is $(1-F-a/2)ert/2$ while in N installment payments cases are $(1-\Phi-\alpha/N)e^{rt/N}$, $(1-\Phi-2\alpha/N)e^{2rt/N}$, $(1-\Phi-(N-1)\alpha/N)e^{rt(N-1)/N}$, ..., $(1-\Phi-\alpha)e^{rt}$. Now, I consider two installment payments case.

With the assumptions listed above except no depreciation, the value of a European-style buyer's call option on a call at time s can be shown as

$$C_i(C_s, s; C_s^*) = S_s e^{-r(s+\tau)} N_2(a_1, b_1; (s/(s+\tau))^{1/2}) - X e^{-r(s+\tau)} N_2(a_2, b_2; (s/(s+\tau))^{1/2}) - e^{-rs} C_s^* N_1(b_2) \dots\dots\dots (7)$$

where $s = \tau / 2$ (in the middle of the construction stage);

$$a_1 = [\ln(S/X) + (0.5\sigma^2)(s+\tau)] / [\sigma(s+\tau)^{1/2}]; a_2 = a_1 - \sigma(s+\tau)^{1/2};$$

$b_1 = [\ln(S/S_1^*) + (0.5\sigma^2)s] / \sigma s^{1/2}; b_2 = b_1 - \sigma s^{1/2}$, and $N_1(\cdot)$ and $N_2(\cdot)$ are the cumulative univariate and bivariate unit normal density functions respectively. The term $N_2(a_1, b_1; [s/(s+\tau)]^{1/2})$ is the delta value of the call option on a call option. It describes the call option price movement for a small change in the existing house price. The term $N_1(b_2)$ is the probability that the existing house price will exceed the critical existing house price at time s . The term $N_2(a_2, b_2; [s/(s+\tau)]^{1/2})$ is the probability that the existing house price will exceed S_s^* at time s and the exercise price X at time $s+\tau$. By applying arbitrage argument, I can derive the pricing model for the pre-sales housing with buyer's call option as

$$F(t) = S(t) + C_i(C_s, s; C_s^*) + S(t) [2-2\Phi-\alpha/2](e^{r\tau/2}-1) - e^{ct} \dots\dots\dots (8)$$

From equation(8), one can see that the buyer's call option makes the pre-sale housing price higher than that without this option.

With depreciation rate of δ , initial payment (Φ) and 2 installment payments, the value of a European-style buyer's call option on a call will be

$$C(C_s, s; C_s^*) = S_s e^{(\delta-r)(s+\tau)} N_2(a_1, b_1; (s/(s+\tau))^{1/2}) - X e^{-r(s+\tau)} N_2(a_2, b_2; (s/(s+\tau))^{1/2}) - e^{-rs} C_s^* N_1(b_2) \dots\dots\dots (9)$$

where $s = \tau / 2$ (in the middle of the construction stage);

$$a_1 = [\ln(S/X) + (\delta+0.5\sigma^2)(s+\tau)] / [\sigma(s+\tau)^{1/2}]; a_2 = a_1 - \sigma(s+\tau)^{1/2};$$

$$b_1 = [\ln(S/S_1^*) + (\delta+0.5\sigma^2)s] / \sigma s^{1/2}; b_2 = b_1 - \sigma s^{1/2}.$$

By applying arbitrage argument, I can derive the pricing model for the pre-sale housing as

$$F(t) = S(t) + C_i(C_s, s; C_s^*) + S(t) [e^{\delta\tau} - e^{ct} + (2-2\Phi-\alpha/2)(e^{r\tau/2}-1)] \dots\dots\dots (10)$$

where $C(C_s, s; C_s^*)$ is defined as equation (9). Equation (10) is a generalized pricing model for evaluating the pre-sale housing and hence Shyy's [1992] model is its special case. With the same argument, I also can derive the pricing model for pre-sale housing under N installment payments case by considering $(N-1)$ compound options solved numerically (Geske[1979]).

3. CONCLUDING REMARKS

In this note, I have extended Shyy's (1992) model and applied the compound option model in pricing the pre-sale housing. By using simple arbitrage arguments to relate the pre-sales and existing house prices, I apply the carrying charge model as well as compound option models to price the pre-sale housing under the assumption of same characteristics of pre-sale and existing houses and show that the price of pre-sale housing could be higher or lower than that of the existing house. It will depend on the macroeconomic factors used to explain the level of interest rate for the time deposit, the rental rate, the depreciation rate and even the economic cycle. By considering the buyer's call option, this generalized model might provide some useful evidence to complement the current findings of empirical estimates.

REFERENCES

- Black, F., & M. Scholes
 1973 "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, May/June: 637-659.
- Chang, Chin-Oh, & Steven C. L. Farr
 1993 "A Study of Real Estate Transaction Price," *Journal of Housing Study* 1: 75-97.
- Chang, Chin-Oh, & Charles W.R. Ward
 1993 "Forward Pricing and the Housing Market: the Pre-Sales Housing system in Taiwan," *Journal of Property Research* 10: 217-227.
- Chang, Li-Chi
 1994 "Applying the Theoretical Relationship Between Forward and Spot Prices to the Pre-Sale and Existing Housing Markets - A Case in Taipei," *Journal of Housing Study* 2: 67-85.
- Geske, Robert
 1979 "The Valuation of Compound Options," *Journal of Financial Economics* 7, March: 63-81.
- Hwang, Dar-Yeh
 1994 "Valuation of the Pre-Sale Housing in Taiwan Housing Markets," *Conference Paper of the Chinese Finance Association*: 85-109.
- Shyy, Gang
 1992 "Real Estate Pricing in A Forward Market," *Journal of Management Science*, Vol. 9, No.1: 31-37.